HL Paper 1

A point P, relative to an origin O, has position vector $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix}$, $s \in \mathbb{R}$.

Find the minimum length of $\overrightarrow{\mathrm{OP}}$.

Markscheme

METHOD 1

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{(1+s)^2 + (3+2s)^2 + (1-s)^2} \quad \left(= \sqrt{6s^2 + 12s + 11}\right)$ ***A1***

Note: Award ***A1*** if the square of the distance is found.

EITHER

attempt to differentiate: $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right|^2 \quad (= 12s + 12)$ ***M1***

attempting to solve $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right|^2 = 0 \quad \text{for } s$ ***(M1)***

$s = -1$ ***(A1)***

OR

attempt to differentiate: $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right| \quad \left(= \frac{6s+6}{\sqrt{6s^2+12s+11}}\right)$ ***M1***

attempting to solve $\frac{\mathrm{d}}{\mathrm{d}s}\left|\overrightarrow{\mathrm{OP}}\right| = 0 \quad \text{for } s$ ***(M1)***

$s = -1$ ***(A1)***

OR

attempt at completing the square: $\left(\left|\overrightarrow{\mathrm{OP}}\right|^2 = 6(s+1)^2 + 5\right)$ ***M1***

minimum value ***(M1)***

occurs at $s = -1$ ***(A1)***

THEN

the minimum length of $\overrightarrow{\mathrm{OP}}$ is $\sqrt{5}$ ***A1***

METHOD 2

the length of $\overrightarrow{\mathrm{OP}}$ is a minimum when $\overrightarrow{\mathrm{OP}}$ is perpendicular to $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ ***(R1)***

$\begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 0$ ***A1*** attempting to solve $1 + s + 6 + 4s - 1 + s = 0 \quad (6s + 6 = 0) \quad \text{for } s$ ***(M1)***

$s = -1$ ***(A1)***

$\left|\overrightarrow{\mathrm{OP}}\right| = \sqrt{5}$ ***(A1)***

[5 marks]

Examiners report

Generally well done. But there was a significant minority who didn't realise that they had to use calculus or completion of squares to minimise the length. Trying random values of s gained no marks. A number of candidates wasted time showing that their answer gave a minimum rather than a maximum value of the length.

Three distinct non-zero vectors are given by $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$, and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

If $\overrightarrow{\mathrm{OA}}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$ and $\overrightarrow{\mathrm{OB}}$ is perpendicular to $\overrightarrow{\mathrm{CA}}$, show that $\overrightarrow{\mathrm{OC}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$.

Markscheme

$\overrightarrow{\mathrm{BC}} = \boldsymbol{c} - \boldsymbol{b}$

$\overrightarrow{\mathrm{CA}} = \boldsymbol{a} - \boldsymbol{c}$

$\Rightarrow \boldsymbol{a}\cdot(\boldsymbol{c} - \boldsymbol{b}) = 0$ ***M1***

and $\boldsymbol{b}\cdot(\boldsymbol{a} - \boldsymbol{c}) = 0$ ***M1***

$\Rightarrow \boldsymbol{a}\cdot\boldsymbol{c} = \boldsymbol{a}\cdot\boldsymbol{b}$ ***A1***

and $\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{b}\cdot\boldsymbol{c}$ ***A1***

$\Rightarrow \boldsymbol{a}\cdot\boldsymbol{c} = \boldsymbol{b}\cdot\boldsymbol{c}$ ***M1***

$\Rightarrow \boldsymbol{b}\cdot\boldsymbol{c} - \boldsymbol{a}\cdot\boldsymbol{c} = 0$

$\boldsymbol{c}\cdot(\boldsymbol{b} - \boldsymbol{a}) = 0$ ***A1***

$\Rightarrow \overrightarrow{\mathrm{OC}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$, as $\boldsymbol{b} \neq \boldsymbol{a}$. ***AG***

[6 marks]

Examiners report

Only the better candidates were able to make significant progress with this question. Many candidates understood how to begin the question, but did not see how to progress to the last stage. On the whole the candidates' use of notation in this question was poor.

Consider the vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{a} + \boldsymbol{b}$. Show that if $|\boldsymbol{a}| = |\boldsymbol{b}|$ then $(\boldsymbol{a} + \boldsymbol{b})\cdot(\boldsymbol{a} - \boldsymbol{b}) = 0$. Comment on what this tells us about the parallelogram OACB.

Markscheme

$(\boldsymbol{a}+\boldsymbol{b})\cdot(\boldsymbol{a}-\boldsymbol{b}) = \boldsymbol{a}\cdot\boldsymbol{a}+\boldsymbol{b}\cdot\boldsymbol{a}-\boldsymbol{a}\cdot\boldsymbol{b}-\boldsymbol{b}\cdot\boldsymbol{b}$ ***M1***

$= \boldsymbol{a}\cdot\boldsymbol{a}-\boldsymbol{b}\cdot\boldsymbol{b}$ ***A1***

$= |\boldsymbol{a}|^2 - |\boldsymbol{b}|^2 = 0$ since $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***A1***

the **diagonals** are perpendicular ***R1***

Note: Accept geometric proof, awarding ***M1*** for recognizing OACB is a rhombus, ***R1*** for a clear indication that $(\boldsymbol{a}+\boldsymbol{b})$ and $(\boldsymbol{a}-\boldsymbol{b})$ are the diagonals, ***A1*** for stating that diagonals cross at right angles and ***A1*** for "hence dot product is zero".

Accept solutions using components in 2 or 3 dimensions.

[4 marks]

Examiners report

Many candidates found this more abstract question difficult. While there were some correct statements, they could not "show" the result that was asked. Some treated the vectors as scalars and notation was poor, making it difficult to follow what they were trying to do. Very few candidates realized that $a-b$ and $a+b$ were the diagonals of the parallelogram which prevented them from identifying the significance of the result proved. A number of candidates were clearly not aware of the difference between scalars and vectors.

Consider the plane with equation $4x-2y-z=1$ and the line given by the parametric equations

$x = 3-2\lambda$

$y = (2k-1)+\lambda$

$z = -1+k\lambda.$

Given that the line is perpendicular to the plane, find

(a) the value of k;

(b) the coordinates of the point of intersection of the line and the plane.

Markscheme

(a) $\boldsymbol{a} = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} \perp$ to the plane $\boldsymbol{e} = \begin{pmatrix} -2 \\ 1 \\ k \end{pmatrix}$ is parallel to the line ***(A1)(A1)***

Note: Award ***A1*** for each correct vector written down, even if not identified.

line $\perp$ plane $\Rightarrow$ $\boldsymbol{e}$ parallel to $\boldsymbol{a}$

since $\begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} = t\begin{pmatrix} -2 \\ 1 \\ k \end{pmatrix} \Rightarrow k = \frac{1}{2}$ ***(M1)A1***

(b) $4(3-2\lambda)-2\lambda-\left(-1+\frac{1}{2}\lambda\right) = 1$ ***(M1)(A1)***

Note: ***FT*** their value of k as far as possible.

$\lambda = \frac{8}{7}$ ***A1***
$\mathrm{P}\left(\frac{5}{7}, \frac{8}{7}, -\frac{3}{7}\right)$ ***A1***

[8 marks]

Examiners report

Solutions to this question were often disappointing. In (a), some candidates found the value of k, incorrectly, by taking the scalar product of the normal vector to the plane and the direction of the line. Such candidates benefitted partially from follow through in (b) but not fully because their line turned out to be parallel to the plane and did not intersect it.

Two boats, A and B , move so that at time t hours, their position vectors, in kilometres, are $\boldsymbol{r}_A = (9t)\boldsymbol{i} + (3 - 6t)\boldsymbol{j}$ and $\boldsymbol{r}_B = (7 - 4t)\boldsymbol{i} + (7t - 6)\boldsymbol{j}$.

a. Find the coordinates of the common point of the paths of the two boats. [4]

b. Show that the boats do not collide. [2]

Markscheme

a. **METHOD 1**

$9t_A = 7 - 4t_B$ and
$3 - 6t_A = -6 + 7t_B$ ***M1A1***
solve simultaneously
$t_A = \frac{1}{3},\ t_B = 1$ ***A1***
Note: Only need to see one time for the ***A1***.

therefore meet at (3, 1) ***A1***

[4 marks]

METHOD 2

path of A is a straight line: $y = -\frac{2}{3}x + 3$ ***M1A1***

Note: Award ***M1*** for an attempt at simultaneous equations.

path of B is a straight line: $y = -\frac{7}{4}x + \frac{25}{4}$ ***A1***

$-\frac{2}{3}x + 3 = -\frac{7}{4}x + \frac{25}{4}\ (\Rightarrow x = 3)$

so the common point is (3, 1) ***A1***

[4 marks]

b. **METHOD 1**

boats do not collide because the two times $\left(t_A = \frac{1}{3},\ t_B = 1\right)$ ***(A1)*** are different ***R1***

[2 marks]

METHOD 2

for boat A, $9t = 3 \Rightarrow t = \frac{1}{3}$ and for boat B, $7 - 4t = 3 \Rightarrow t = 1$

times are different so boats do not collide ***R1AG***

[2 marks]

Examiners report

a. This was probably the least accessible question from section A. Most started by using the same value of *t* in attempting to find the common point, and so scored no marks. There were a number of very good candidates who set different parameters for *t* and correctly obtained (3,1) . There was slightly better understanding shown in part b), though some argued that the boats did not collide because their times were different, yet then provided incorrect times, or even no times at all.

b. This was probably the least accessible question from section A. Most started by using the same value of *t* in attempting to find the common point, and so scored no marks. There were a number of very good candidates who set different parameters for *t* and correctly obtained (3,1) . There was slightly better understanding shown in part b), though some argued that the boats did not collide because their times were different, yet then provided incorrect times, or even no times at all.

O, A, B and C are distinct points such that $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

It is given that $\boldsymbol{c}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$ and $\boldsymbol{b}$ is perpendicular to $\overrightarrow{\mathrm{AC}}$.

Prove that $\boldsymbol{a}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$.

Markscheme

$\boldsymbol{c} \bullet (\boldsymbol{b} - \boldsymbol{a}) = 0$ ***M1***

Note: Allow $c \bullet \overrightarrow{\mathrm{AB}} = 0$ or similar for ***M1***.

$\boldsymbol{c} \bullet \boldsymbol{b} = \boldsymbol{c} \bullet \boldsymbol{a}$ ***A1***

$\boldsymbol{b} \bullet (\boldsymbol{c} - \boldsymbol{a}) = 0$

$\boldsymbol{b} \bullet \boldsymbol{c} = \boldsymbol{b} \bullet \boldsymbol{a}$ ***A1***

$\boldsymbol{c} \bullet \boldsymbol{a} = \boldsymbol{b} \bullet \boldsymbol{a}$ ***M1***

$(\boldsymbol{c} - \boldsymbol{b}) \bullet \boldsymbol{a} = 0$ ***A1***

hence $\boldsymbol{a}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$ ***AG***

Note: Only award the final ***A1*** if a dot is used throughout to indicate scalar product.

Condone any lack of specific indication that the letters represent vectors.

[5 marks]

Examiners report

This was generally poorly done. The recent syllabus change refers to ‘proof of geometrical properties using vectors’ and this is clearly a topic candidates are not entirely clear with at the moment. Despite the question clearly being written as a vector question some students tried to use a geometrical approach, assuming it was two-dimensional. Many did not seem to realise that vectors being perpendicular implies that their scalar product is zero.

The points A, B, C and D have position vectors ***a***, ***b***, ***c*** and ***d***, relative to the origin O.

It is given that $\vec{\mathrm{AB}} = \vec{\mathrm{DC}}$.

The position vectors $\vec{\mathrm{OA}}$, $\vec{\mathrm{OB}}$, $\vec{\mathrm{OC}}$ and $\vec{\mathrm{OD}}$ are given by

$\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$

$\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k}$

$\boldsymbol{c} = q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$

$\boldsymbol{d} = -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}$

where *p* , *q* and *r* are constants.

The point where the diagonals of ABCD intersect is denoted by M.

The plane Π cuts the *x*, *y* and *z* axes at X , Y and Z respectively.

a.i. Explain why ABCD is a parallelogram. [1]

a.ii. Using vector algebra, show that $\vec{\mathrm{AD}} = \vec{\mathrm{BC}}$. [3]

b. Show that *p* = 1, *q* = 1 and *r* = 4. [5]

c. Find the area of the parallelogram ABCD. [4]

d. Find the vector equation of the straight line passing through M and normal to the plane Π containing ABCD. [4]

e. Find the Cartesian equation of Π. [3]

f.i. Find the coordinates of X, Y and Z. [2]

f.ii. Find YZ. [2]

Markscheme

a.i. a pair of opposite sides have equal length and are parallel ***R1***

hence ABCD is a parallelogram ***AG***

[1 mark]

a.ii. attempt to rewrite the given information in vector form ***M1***

b – ***a*** = ***c*** – ***d*** ***A1***

rearranging ***d*** – ***a*** = ***c*** – ***b*** ***M1***

hence $\vec{\mathrm{AD}} = \vec{\mathrm{BC}}$ ***AG***

Note: Candidates may correctly answer part i) by answering part ii) correctly and then deducing there are two pairs of parallel sides.

[3 marks]

b. **EITHER**

use of $\vec{\mathrm{AB}} = \vec{\mathrm{DC}}$ ***(M1)***

$$\begin{pmatrix} 2 \\ -3 \\ p+3 \end{pmatrix} = \begin{pmatrix} q+1 \\ 1-r \\ 4 \end{pmatrix}$$ ***A1A1***

OR

use of $\vec{\mathrm{AD}} = \vec{\mathrm{BC}}$ ***(M1)***

$$\begin{pmatrix} -2 \\ r-2 \\ 1 \end{pmatrix} = \begin{pmatrix} q-3 \\ 2 \\ 2-p \end{pmatrix}$$ ***A1A1***

THEN

attempt to compare coefficients of ***i***, ***j***, and ***k*** in their equation or statement to that effect ***M1***

clear demonstration that the given values satisfy their equation ***A1***
p = 1, *q* = 1, *r* = 4 ***AG***

[5 marks]

c. attempt at computing $\vec{\mathrm{AB}} \times \vec{\mathrm{AD}}$ (or equivalent) ***M1***

$$\begin{pmatrix} -11 \\ -10 \\ -2 \end{pmatrix}$$ ***A1***

area $= \left|\vec{\mathrm{AB}} \times \vec{\mathrm{AD}}\right| \left(= \sqrt{225}\right)$ ***(M1)***

= 15 ***A1***

[4 marks]

d. valid attempt to find $\vec{\mathrm{OM}} = \left(\frac{1}{2}(a+c)\right)$ ***(M1)***

$$\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix}$$ ***A1***

the equation is

r = $\begin{pmatrix} 1 \\ \frac{3}{2} \\ -\frac{1}{2} \end{pmatrix} + t\begin{pmatrix} 11 \\ 10 \\ 2 \end{pmatrix}$ or equivalent ***M1A1***

Note: Award maximum ***M1A0*** if '***r*** = …' (or equivalent) is not seen.

[4 marks]

e. attempt to obtain the equation of the plane in the form *ax* + *by* + *cz* = *d* ***M1***

$11x + 10y + 2z = 25$ ***A1A1***

Note: ***A1*** for right hand side, ***A1*** for left hand side.

[3 marks]

f.i. putting two coordinates equal to zero ***(M1)***

$\mathrm{X}\left(\frac{25}{11},\ 0,\ 0\right),\ \mathrm{Y}\left(0,\ \frac{5}{2},\ 0\right),\ \mathrm{Z}\left(0,\ 0,\ \frac{25}{2}\right)$ ***A1***

[2 marks]

f.ii. $\mathrm{YZ} = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{25}{2}\right)^2}$ ***M1***

$= \sqrt{\frac{325}{2}} \left(= \frac{5\sqrt{104}}{4} = \frac{5\sqrt{26}}{2}\right)$ ***A1***

[4 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f.i. [N/A]
f.ii. [N/A]

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a. Show that the points $\mathrm{O}(0,\ 0,\ 0),\ \mathrm{A}(6,\ 0,\ 0), \mathrm{B}(6,\ -\sqrt{24},\ \sqrt{12}), \mathrm{C}(0,\ -\sqrt{24},\ \sqrt{12})$ form a square. [3]

b. Find the coordinates of M, the mid-point of [OB]. [1]

c. Show that an equation of the plane *Π*, containing the square OABC, is $y + \sqrt{2}z = 0$. [3]

d. Find a vector equation of the line L, through M, perpendicular to the plane *Π*. [3]

e. Find the coordinates of D, the point of intersection of the line L with the plane whose equation is $y = 0$. [3]

f. Find the coordinates of E, the reflection of the point D in the plane *Π*. [3]

g. (i) Find the angle $\mathrm{O\hat{D}A}$. [6]

(ii) State what this tells you about the solid OABCDE.

Markscheme

a. $\left|\overrightarrow{\mathrm{OA}}\right| = \left|\overrightarrow{\mathrm{CB}}\right| = \left|\overrightarrow{\mathrm{OC}}\right| = \left|\overrightarrow{\mathrm{AB}}\right| = 6$ (therefore a rhombus) ***A1A1***

Note: Award ***A1*** for two correct lengths, ***A2*** for all four.

Note: Award ***A1A0*** for $\overrightarrow{\text{OA}} = \overrightarrow{\text{CB}} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ or $\overrightarrow{\text{OC}} = \overrightarrow{AB} = \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix}$ if no magnitudes are shown.

$\overrightarrow{\text{OA}}\ \text{g}\overrightarrow{\text{OC}} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \text{g} \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix} = 0$ (therefore a square) ***A1***

Note: Other arguments are possible with a minimum of three conditions.

[3 marks]

b. $\text{M}\left(3,\ -\frac{\sqrt{24}}{2},\ \frac{\sqrt{12}}{2}\right)\left(=\left(3,\ -\sqrt{6},\ \sqrt{3}\right)\right)$ ***A1***

[1 mark]

c. **METHOD 1**

$\overrightarrow{\text{OA}} \times \overrightarrow{\text{OC}} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -\sqrt{24} \\ \sqrt{12} \end{pmatrix} = \begin{pmatrix} 0 \\ -6\sqrt{12} \\ -6\sqrt{24} \end{pmatrix}\left(= \begin{pmatrix} 0 \\ -12\sqrt{3} \\ -12\sqrt{6} \end{pmatrix}\right)$ ***M1A1***

Note: Candidates may use other pairs of vectors.

equation of plane is $-6\sqrt{12}y - 6\sqrt{24}z = d$
any valid method showing that $d = 0$ ***M1***
$\Pi : y + \sqrt{2}z = 0$ ***AG***

METHOD 2
equation of plane is $ax + by + cz = d$
substituting O to find $d = 0$ ***(M1)***
substituting two points (A, B, C or M) ***M1***
eg
$6a = 0,\ -\sqrt{24}b + \sqrt{12}c = 0$ ***A1***
$\Pi : y + \sqrt{2}z = 0$ ***AG***
[3 marks]

d. $\boldsymbol{r} = \begin{pmatrix} 3 \\ -\sqrt{6} \\ \sqrt{3} \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ \sqrt{2} \end{pmatrix}$ ***A1A1A1***

Note: Award ***A1*** for ***r*** = , ***A1A1*** for two correct vectors.

[3 marks]

e. Using $y = 0$ to find λ ***M1***

Substitute their λ into their equation from part (d) ***M1***
D has coordinates $\left(3, 0, 3\sqrt{3}\right)$ ***A1***

[3 marks]

f. λ for point E is the negative of the λ for point D ***(M1)***

Note: Other possible methods may be seen.

E has coordinates $\left(3,\ -2\sqrt{6},\ -\sqrt{3}\right)$ ***A1A1***

Note: Award ***A1*** for each of the *y* and *z* coordinates.

[3 marks]

g. (i) $\overrightarrow{\rm DA}\ {\rm g}\overrightarrow{\rm DO} = \begin{pmatrix} 3 \\ 0 \\ -3\sqrt{3} \end{pmatrix} {\rm g} \begin{pmatrix} -3 \\ 0 \\ -3\sqrt{3} \end{pmatrix} = 18$ ***M1A1***

$\cos {\rm O\hat{D}A} = \frac{18}{\sqrt{36}\sqrt{36}} = \frac{1}{2}$ ***M1***

hence ${\rm O\hat{D}A} = 60^\circ$ ***A1***

Note: Accept method showing OAD is equilateral.

(ii) OABCDE is a regular octahedron (accept equivalent description) ***A2***

Note: ***A2*** for saying it is made up of 8 equilateral triangles
Award ***A1*** for two pyramids, ***A1*** for equilateral triangles.
(can be either stated or shown in a sketch – but there must be clear indication the triangles are equilateral)

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

(a) Show that the two planes

$$\pi_1 : x + 2y - z = 1$$

$$\pi_2 : x + z = -2$$

are perpendicular.

(b) Find the equation of the plane π_3 that passes through the origin and is perpendicular to both π_1 and π_2.

Markscheme

(a) for using normal vectors ***(M1)***

$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 1 - 1 = 0$ ***M1A1***

hence the two planes are perpendicular ***AG***

(b) **METHOD 1**

EITHER

$\begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 2\mathbf{i} - 2\boldsymbol{j} - 2\boldsymbol{k}$ ***M1A1***

OR

if $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is normal to π_3, then

$a + 2b - c = 0$ and $a + c = 0$ ***M1***

a solution is $a = 1, b = -1, c = -1$ ***A1***

THEN

π_3 has equation $x - y - z = d$ ***(M1)***

as it goes through the origin, $d = 0$ so π_3 has equation $x - y - z = 0$ ***A1***

Note: The final ***(M1)A1*** are independent of previous working.

METHOD 2

$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ***A1(A1)A1A1***

[7 marks]

Examiners report

Although many candidates were successful in answering this question, a surprising number showed difficulties in working with normal vectors. In part (b) there were several candidates who found the cross product of the vectors but were unable to use it to write the equation of the plane.

The following system of equations represents three planes in space.

$$x + 3y + z = -1$$
$$x + 2y - 2z = 15$$
$$2x + y - z = 6$$

Find the coordinates of the point of intersection of the three planes.

Markscheme

EITHER

eliminating a variable, x, for example to obtain $y + 3z = -16$ and $-5y - 3z = 8$ ***M1A1*** attempting to find the value of one variable ***M1***

point of intersection is $(-1,\ 2,\ -6)$ ***A1A1A1***

OR

attempting row reduction of relevant matrix, *eg*. $\left(\begin{array}{ccc|c} 2 & 1 & -1 & 6 \\ 1 & 3 & 1 & -1 \\ 1 & 2 & -2 & 15 \end{array}\right)$ ***M1***

correct matrix with two zeroes in a column, *eg*. $\left(\begin{array}{ccc|c} 2 & 1 & -1 & 6 \\ 0 & 5 & 3 & -8 \\ 0 & 1 & 3 & -16 \end{array}\right)$ ***A1***

further attempt at reduction ***M1***

point of intersection is $(-1,\ 2,\ -6)$ ***A1A1A1***

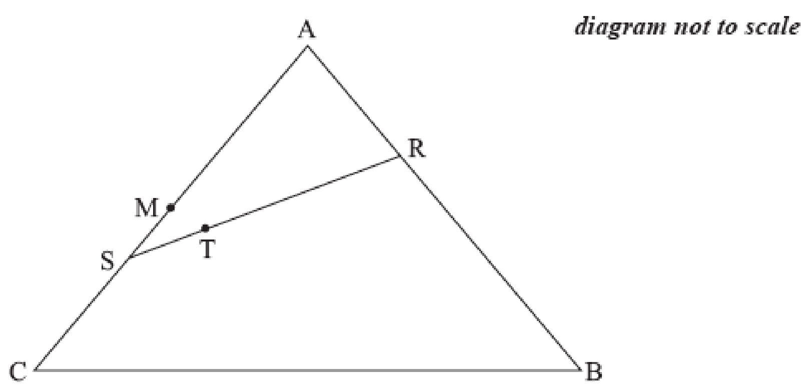
Note: Allow solution expressed as $x = -1,\ y = 2,\ z = -6$ for final ***A*** marks.

[6 marks]

Examiners report

This provided a generally easy start for many candidates. Most successful candidates obtained their answer through row reduction of a suitable matrix. Those choosing an alternative method often made slips in their algebra.

The position vectors of the points A, B and C are a, b and c respectively, relative to an origin O. The following diagram shows the triangle ABC and points M, R, S and T.



M is the midpoint of $[AC]$.

R is a point on $[AB]$ such that $\overrightarrow{AR} = \frac{1}{3}\overrightarrow{AB}$.

S is a point on $[AC]$ such that $\overrightarrow{AS} = \frac{2}{3}\overrightarrow{AC}$.

T is a point on $[RS]$ such that $\overrightarrow{RT} = \frac{2}{3}\overrightarrow{RS}$.

a. (i) Express $\overrightarrow{AM}$ in terms of a and c. [4]

(ii) Hence show that $\overrightarrow{\mathrm{BM}} = \frac{1}{2}a - b + \frac{1}{2}c$.

b. (i) Express $\overrightarrow{\mathrm{RA}}$ in terms of a and b. [5]

(ii) Show that $\overrightarrow{RT} = -\frac{2}{9}a - \frac{2}{9}b + \frac{4}{9}c$.

c. Prove that T lies on $[BM]$. [5]

Markscheme

a. (i) $\overrightarrow{\mathrm{AM}} = \frac{1}{2}\overrightarrow{\mathrm{AC}}$ ***(M1)***

$= \frac{1}{2}(c - a)$ ***A1***

(ii) $\overrightarrow{\mathrm{BM}} = \overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AM}}$ ***M1***

$= a - b + \frac{1}{2}(c - a)$ ***A1***

$\overrightarrow{\mathrm{BM}} = \frac{1}{2}a - b + \frac{1}{2}c$ ***AG***

[4 marks]

b. (i) $\overrightarrow{\mathrm{RA}} = \frac{1}{3}\overrightarrow{\mathrm{BA}}$

$= \frac{1}{3}(\boldsymbol{a} - \boldsymbol{b})$ ***A1***

(ii) $\overrightarrow{\mathrm{RT}} = \frac{2}{3}\overrightarrow{\mathrm{RS}}$

$= \frac{2}{3}\left(\overrightarrow{\mathrm{RA}} + \overrightarrow{\mathrm{AS}}\right)$ ***(M1)***

$= \frac{2}{3}\left(\frac{1}{3}(a - b) + \frac{2}{3}(c - a)\right)$ or equivalent. ***A1A1***

$= \frac{2}{9}(a - b) + \frac{4}{9}(c - a)$ ***A1***

$\overrightarrow{\mathrm{RT}} = -\frac{2}{9}a - -\frac{2}{9}b + \frac{4}{9}c$ ***AG***

[5 marks]

c. $\overrightarrow{\mathrm{BT}} = \overrightarrow{\mathrm{BR}} + \overrightarrow{\mathrm{RT}}$

$= \frac{2}{3}\overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{RT}}$ ***(M1)***

$= \frac{2}{3}a - \frac{2}{3}b - \frac{2}{9}a - \frac{2}{9}b + \frac{4}{9}c$ ***A1***

$\overrightarrow{\mathrm{BT}} = \frac{8}{9}\left(\frac{1}{2}a - b + \frac{1}{2}c\right)$ ***A1***

point B is common to $\overrightarrow{\mathrm{BT}}$ and $\overrightarrow{\mathrm{BM}}$ and $\overrightarrow{\mathrm{BT}} = \frac{8}{9}\overrightarrow{\mathrm{BM}}$ ***R1R1***

so T lies on $[BM]$ ***AG***

[5 marks]

Total [14 marks]

Examiners report

a. A fairly straightforward question for candidates confident in the use of and correct notation for relative position vectors. Sign errors were the most common, but the majority of candidates did not gain all the reasoning marks for part (c). In particular, it was necessary to observe that not only were two vectors parallel, but that they had a point in common.

b. A fairly straightforward question for candidates confident in the use of and correct notation for relative position vectors. Sign errors were the most common, but the majority of candidates did not gain all the reasoning marks for part (c). In particular, it was necessary to observe that not only were two vectors parallel, but that they had a point in common.

c. A fairly straightforward question for candidates confident in the use of and correct notation for relative position vectors. Sign errors were the most common, but the majority of candidates did not gain all the reasoning marks for part (c). In particular, it was necessary to observe that not only were two vectors parallel, but that they had a point in common.

Find the values of x for which the vectors $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix}$ are perpendicular, $0 \leqslant x \leqslant \frac{\pi}{2}$.

Markscheme

perpendicular when $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix} = 0$ ***(M1)***

$\Rightarrow -1 + 4\sin x\cos x = 0$ ***A1***

$\Rightarrow \sin 2x = \frac{1}{2}$ ***M1***

$\Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ ***A1A1***

Note: Accept answers in degrees.

[5 marks]

Examiners report

Most candidates realised that the scalar product should be used to solve this problem and many obtained the equation $4\sin x\cos x = 1$. Candidates who failed to see that this could be written as $\sin 2x = 0.5$ usually made no further progress. The majority of those candidates who used this double angle formula carried on to obtain the solution $\frac{\pi}{12}$ but few candidates realised that $\frac{5\pi}{12}$ was also a solution.

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 2\boldsymbol{k}$.

a. Find $\boldsymbol{a} \times \boldsymbol{b}$. [2]

b. Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and passing through the point $(1,\ 0,\ -1)$. [3]

Markscheme

a. $\boldsymbol{a} \times \boldsymbol{b} = -12\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ ***(M1)A1***

[2 marks]

b. **METHOD 1**

$-12x - 2y - 3z = d$ ***M1***

$-12 \times 1 - 2 \times 0 - 3(-1) = d$ ***(M1)***

$\Rightarrow d = -9$ ***A1***

$-12x - 2y - 3z = -9$ (or $12x + 2y + 3z = 9$)

METHOD 2

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} -12 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} -12 \\ -2 \\ -3 \end{pmatrix}$$ ***M1A1***

$-12x - 2y - 3z = -9$ (or $12x + 2y + 3z = 9$) ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The points A(1, 2, 1) , B(−3, 1, 4) , C(5, −1, 2) and D(5, 3, 7) are the vertices of a tetrahedron.

a. Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

b. Find the Cartesian equation of the plane $\prod$ that contains the face ABC. [4]

Markscheme

a. $\overrightarrow{AB} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$ ***A1A1***

Note: Accept row vectors.

[2 marks]

b. $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -4 & -1 & 3 \\ 4 & -3 & 1 \end{vmatrix} = \begin{pmatrix} 8 \\ 16 \\ 16 \end{pmatrix}$ ***M1A1***

normal $\boldsymbol{n} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ so $\boldsymbol{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***(M1)***

$x + 2y + 2z = 7$ ***A1***

Note: If attempt to solve by a system of equations:
Award ***A1*** for 3 correct equations, ***A1*** for eliminating a variable and ***A2*** for the correct answer.

[4 marks]

Examiners report

a. Most candidates attempted this question and scored at least a few marks in (a) and (b). Part (c) was more challenging to many candidates who were unsure how to find the required distance. Part (d) was attempted by many candidates some of whom benefited from follow through marks due to errors in previous parts. However, many candidates failed to give the correct answer to this question due to the use of the simplified vector found in (b) showing little understanding of the role of the magnitude of this vector. Part (e) was poorly answered. Overall, this question was not answered to the expected level, showing that many candidates have difficulties with vectors and are unable to answer even standard questions on this topic.

b. Most candidates attempted this question and scored at least a few marks in (a) and (b). Part (c) was more challenging to many candidates who were unsure how to find the required distance. Part (d) was attempted by many candidates some of whom benefited from follow through marks due to errors in previous parts. However, many candidates failed to give the correct answer to this question due to the use of the simplified vector found in (b) showing little understanding of the role of the magnitude of this vector. Part (e) was poorly answered. Overall, this question was not answered to the expected level, showing that many candidates have difficulties with vectors and are unable to answer even standard questions on this topic.

Consider the points $\text{A}(1, 0, 0)$, $\text{B}(2, 2, 2)$ and $\text{C}(0, 2, 1)$.

A third plane Π_3 is defined by the Cartesian equation $16x + \alpha y - 3z = \beta$.

a. Find the vector $\overrightarrow{\text{CA}} \times \overrightarrow{\text{CB}}$. [4]

b. Find an exact value for the area of the triangle ABC. [3]

c. Show that the Cartesian equation of the plane Π_1, containing the triangle ABC, is $2x + 3y - 4z = 2$. [3]

d. A second plane Π_2 is defined by the Cartesian equation $\Pi_2 : 4x - y - z = 4$. L_1 is the line of intersection of the planes Π_1 and Π_2. [5]
Find a vector equation for L_1.

e. Find the value of α if all three planes contain L_1. [3]

f. Find conditions on α and β if the plane Π_3 does **not** intersect with L_1. [2]

Markscheme

a. $\overrightarrow{\text{CA}} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ ***(A1)***

$\overrightarrow{\text{CB}} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ***(A1)***

Note: If $\overrightarrow{\text{AC}}$ and $\overrightarrow{\text{BC}}$ found correctly award ***(A1) (A0)***.

$\overrightarrow{\text{CA}} \times \overrightarrow{\text{CB}} = \begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 2 & 0 & 1 \end{vmatrix}$ ***(M1)***

$\begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix}$ ***A1***

[4 marks]

b. **METHOD 1**

$\frac{1}{2}\left|\overrightarrow{\text{CA}} \times \overrightarrow{\text{CB}}\right|$ ***(M1)***

$= \frac{1}{2}\sqrt{(-2)^2 + (-3)^2 + 4^2}$ ***(A1)***

$= \frac{\sqrt{29}}{2}$ ***A1***

METHOD 2

attempt to apply $\frac{1}{2}|\text{CA}|\,|\text{CB}|\sin C$ ***(M1)***

$\text{CA.CB} = \sqrt{5}.\sqrt{6}\cos C \Rightarrow \cos C = \frac{1}{\sqrt{30}} \Rightarrow \sin C = \frac{\sqrt{29}}{\sqrt{30}}$ ***(A1)***

area $= \frac{\sqrt{29}}{2}$ ***A1***

[3 marks]

c. **METHOD 1**

$\mathbf{r}.\begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ -3 \\ 4 \end{pmatrix}$ ***M1A1***

$\Rightarrow -2x - 3y + 4z = -2$ ***A1***

$\Rightarrow 2x + 3y - 4z = 2$ ***AG***

METHOD 2

$-2x - 3y + 4z = d$

substituting a point in the plane ***M1A1***

$\text{d} = -2$ ***A1***

$\Rightarrow -2x - 3y + 4z = -2$

$\Rightarrow 2x + 3y - 4z = 2$ ***AG***

Note: Accept verification that all 3 vertices of the triangle lie on the given plane.

[3 marks]

d. **METHOD 1**

$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -4 \\ 4 & -1 & -1 \end{vmatrix} = \begin{pmatrix} -7 \\ -14 \\ -14 \end{pmatrix}$ ***M1A1***

$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$z = 0 \Rightarrow y = 0,\ x = 1$ ***(M1)(A1)***

$L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ ***A1***

Note: Do not award the final ***A1*** if $\mathbf{r} =$ is not seen.

METHOD 2

eliminate 1 of the variables, *eg x* ***M1***

$-7y + 7z = 0$ ***(A1)***

introduce a parameter ***M1***

$\Rightarrow z = \lambda,$

$y = \lambda, x = 1 + \frac{\lambda}{2}$ ***(A1)***

$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ or equivalent ***A1***

Note: Do not award the final ***A1*** if $\mathbf{r} =$ is not seen.

METHOD 3

$z = t$ ***M1***

write *x* and *y* in terms of $t \Rightarrow 4x - y = 4 + t,\ 2x + 3y = 2 + 4t$ or equivalent ***A1***

attempt to eliminate *x* or *y* ***M1***

$x,\ y,\ z$ expressed in parameters

$\Rightarrow z = t,$

$y = t,\ x = 1 + \frac{t}{2}$ ***A1***

$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ or equivalent ***A1***

Note: Do not award the final ***A1*** if $\mathbf{r} =$ is not seen.

[5 marks]

e. **METHOD 1**

direction of the line is perpendicular to the normal of the plane

$\begin{pmatrix} 16 \\ \alpha \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 0$ ***M1A1***

$16 + 2\alpha - 6 = 0 \Rightarrow \alpha = -5$ ***A1***

METHOD 2

solving line/plane simultaneously

$16(1 + \lambda) + 2\alpha\lambda - 6\lambda = \beta$ ***M1A1***

$16 + (10 + 2\alpha)\lambda = \beta$

$\Rightarrow \alpha = -5$ ***A1***

METHOD 3

$\begin{vmatrix} 2 & 3 & -4 \\ 4 & -1 & -1 \\ 16 & \alpha & -3 \end{vmatrix} = 0$ ***M1***

$2(3 + \alpha) - 3(-12 + 16) - 4(4\alpha + 16) = 0$ ***A1***

$\Rightarrow \alpha = -5$ ***A1***

METHOD 4

attempt to use row reduction on augmented matrix ***M1***

to obtain $\left(\begin{array}{ccc|c} 2 & 3 & -4 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & \alpha+5 & \beta-16 \end{array}\right)$ ***A1***

$\Rightarrow \alpha = -5$ ***A1***

[3 marks]

f. $\alpha = -5$ ***A1***

$\beta \neq 16$ ***A1***

[2 marks]

Examiners report

a. Part a) proved an easy start, though a few (weaker) candidates still believe $\overrightarrow{\mathrm{CA}}$ to be $\overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OA}}$.

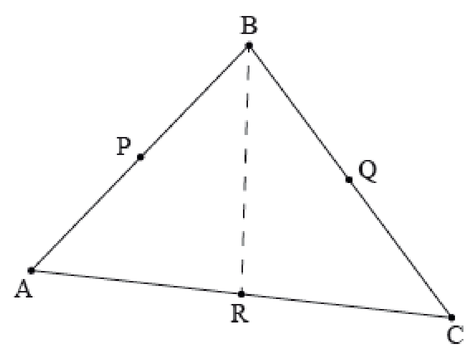
b. Part b) was an easy 3 marks and incorrect answers were rare.

c. Part c) was answered well, though reasoning sometimes seemed sparse, especially given that this was a ‘show that’ question.

d. Part d) proved more challenging, despite being a very standard question. Many candidates gained only 2 marks, either through correctly calculating the direction vector, or by successfully eliminating one of the variables. A number of clear fully correct solutions were seen, though the absence of ‘$\boldsymbol{r}$ =’ is still prevalent, and candidates might be reminded of the correct form for the vector equation of a line.

e. Part e) proved a puzzle for most, though an attempt to use row reduction on an augmented matrix seemed to be the choice way for most successful candidates.

f. Only the very best were able to demonstrate a complete understanding of intersecting planes and thus answer part f) correctly.



Consider the triangle ABC. The points P, Q and R are the midpoints of the line segments [AB], [BC] and [AC] respectively.

Let $\overrightarrow{\mathrm{OA}} = a$, $\overrightarrow{\mathrm{OB}} = b$ and $\overrightarrow{\mathrm{OC}} = c$.

a. Find $\overrightarrow{\mathrm{BR}}$ in terms of a, b and c. [2]

b. (i) Find a vector equation of the line that passes through B and R in terms of a, b and c and a parameter λ. [9]

(ii) Find a vector equation of the line that passes through A and Q in terms of a, b and c and a parameter μ.

(iii) Hence show that $\overrightarrow{\mathrm{OG}} = \frac{1}{3}(a+b+c)$ given that G is the point where [BR] and [AQ] intersect.

c. Show that the line segment [CP] also includes the point G. [3]

d. The coordinates of the points A, B and C are $(1,\ 3,\ 1)$, $(3,\ 7,\ -5)$ and $(2,\ 2,\ 1)$ respectively. [9]

A point X is such that $[GX]$ is perpendicular to the plane ABC.

Given that the tetrahedron $ABCX$ has volume $12\ \text{units}^3$, find possible coordinates of X.

Markscheme

a. $\overrightarrow{\mathrm{BR}} = \overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AR}} \quad \left(= \overrightarrow{\mathrm{BA}} + \frac{1}{2}\overrightarrow{\mathrm{AC}}\right)$ ***(M1)***

$= (a - b) + \frac{1}{2}(c - a)$

$= \frac{1}{2}a - b + \frac{1}{2}c$ ***A1***

[2 marks]

b. (i) $\mathrm{r}_{\mathrm{BR}} = b + \lambda\left(\frac{1}{2}a - b + \frac{1}{2}c\right) \quad \left(= \frac{\lambda}{2}a + (1 - \lambda)b + \frac{\lambda}{2}c\right)$ ***A1A1***

Note: Award ***A1A0*** if the $\mathrm{r} =$ is omitted in an otherwise correct expression/equation.

Do not penalise such an omission more than once.

(ii) $\overrightarrow{\mathrm{AQ}} = -a + \frac{1}{2}b + \frac{1}{2}c$ ***(A1)***

$\mathrm{r}_{\mathrm{AQ}} = a + \mu\left(-a + \frac{1}{2}b + \frac{1}{2}c\right) \quad \left(= (1 - \mu)a + \frac{\mu}{2}b + \frac{\mu}{2}c\right)$ ***A1***

Note: Accept the use of the same parameter in (i) and (ii).

(iii) when $\overrightarrow{\mathrm{AQ}}$ and $\overrightarrow{\mathrm{BP}}$ intersect we will have $\mathrm{r}_{\mathrm{BR}} = \mathrm{r}_{\mathrm{AQ}}$ ***(M1)***

Note: If the same parameters are used for both equations, award at most ***M1M1A0A0M1***.

$\frac{\lambda}{2}a + (1 - \lambda)b + \frac{\lambda}{2}c = (1 - \mu)a + \frac{\mu}{2}b + \frac{\mu}{2}c$

attempt to equate the coefficients of the vectors a, b and c ***M1***

$$\left.\begin{array}{c} \frac{\lambda}{2} = 1 - \mu \\ 1 - \lambda = \frac{\mu}{2} \\ \frac{\lambda}{2} = \frac{\mu}{2} \end{array}\right\}$$ ***(A1)***

$\lambda = \frac{2}{3}$ or $\mu = \frac{2}{3}$ ***A1***

substituting parameters back into one of the equations ***M1***

$\overrightarrow{\mathrm{OG}} = \frac{1}{2} \bullet \frac{2}{3}a + \left(1 - \frac{2}{3}\right)b + \frac{1}{2} \bullet \frac{2}{3}c = \frac{1}{3}(a + b + c)$ ***AG***

Note: Accept solution by verification.

[9 marks]

c. $\overrightarrow{\mathrm{CP}} = \frac{1}{2}a + \frac{1}{2}b - c$ ***(M1)A1*** so we have that $\mathrm{r_{CP}} = c + \beta\left(\frac{1}{2}a + \frac{1}{2}b - c\right)$ and when $\beta = \frac{2}{3}$ the line passes through

the point G (*ie*, with position vector $\frac{1}{3}(a + b + c)$) ***R1***

hence $[AQ]$, $[BR]$ and $[CP]$ all intersect in G ***AG***

[3 marks]

d. $\overrightarrow{\mathrm{OG}} = \frac{1}{3}\left(\begin{pmatrix}1\\3\\1\end{pmatrix} + \begin{pmatrix}3\\7\\-5\end{pmatrix} + \begin{pmatrix}2\\2\\1\end{pmatrix}\right) = \begin{pmatrix}2\\4\\-1\end{pmatrix}$ ***A1***

Note: This independent mark for the vector may be awarded wherever the vector is calculated.

$\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}} = \begin{pmatrix}2\\4\\-6\end{pmatrix} \times \begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}-6\\-6\\-6\end{pmatrix}$ ***M1A1***

$\overrightarrow{\mathrm{GX}} = \alpha\begin{pmatrix}1\\1\\1\end{pmatrix}$ ***(M1)***

volume of Tetrahedron given by $\frac{1}{3} \times \mathrm{Area\ ABC} \times \mathrm{GX}$

$= \frac{1}{3}\left(\frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|\right) \times \mathrm{GX} = 12$ ***(M1)(A1)***

Note: Accept alternative methods, for example the use of a scalar triple product.

$= \frac{1}{6}\sqrt{(-6)^2 + (-6)^2 + (-6)^2} \times \sqrt{\alpha^2 + \alpha^2 + \alpha^2} = 12$ ***(A1)***

$= \frac{1}{6}6\sqrt{3}|\alpha|\sqrt{3} = 12$

$\Rightarrow |\alpha| = 4$ ***A1***

Note: Condone absence of absolute value.

this gives us the position of X as $\begin{pmatrix}2\\4\\-1\end{pmatrix} \pm \begin{pmatrix}4\\4\\4\end{pmatrix}$

$\mathrm{X}(6,\ 8,\ 3)$ or $(-2,\ 0,\ -5)$ ***A1***

Note: Award ***A1*** for either result.

[9 marks]

Total [23 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

Consider the lines l_1 and l_2 defined by

$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and $l_2 : \frac{6-x}{3} = \frac{y-2}{4} = 1 - z$ where a is a constant.

Given that the lines l_1 and l_2 intersect at a point P,

a. find the value of a; [4]

b. determine the coordinates of the point of intersection P. [2]

Markscheme

a. **METHOD 1**

$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} = \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \Rightarrow \begin{cases} x = -3 + \beta \\ y = -2 + 4\beta \\ z = a + 2\beta \end{cases}$ ***M1***

$\frac{6-(-3+\beta)}{3} = \frac{(-2+4\beta)-2}{4} \Rightarrow 4 = \frac{4\beta}{3} \Rightarrow \beta = 3$ ***M1A1***

$\frac{6-(-3+\beta)}{3} = 1 - (a + 2\beta) \Rightarrow 2 = -5 - a \Rightarrow a = -7$ ***A1***

METHOD 2

$\begin{cases} -3 + \beta = 6 - 3\lambda \\ -2 + 4\beta = 4\lambda + 2 \\ a + 2\beta = 1 - \lambda \end{cases}$ ***M1***

attempt to solve ***M1***

$\lambda = 2, \ \beta = 3$ ***A1***

$a = 1 - \lambda - 2\beta = -7$ ***A1***

[4 marks]

b. $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} -3 \\ -2 \\ -7 \end{pmatrix} + 3 \bullet \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 0 \\ 10 \\ -1 \end{pmatrix}$ ***A1***

$\therefore \mathrm{P}(0, \ 10, \ -1)$

[2 marks]

Examiners report

a. [N/A]

b. [N/A]

Consider the plane Π_1, parallel to both lines L_1 and L_2. Point C lies in the plane Π_1.

The line L_3 has vector equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}$.

The plane Π_2 has Cartesian equation $x + y = 12$.

The angle between the line L_3 and the plane Π_2 is 60°.

a. Given the points A(1, 0, 4), B(2, 3, −1) and C(0, 1, − 2) , find the vector equation of the line L_1 passing through the points A and B. [2]

b. The line L_2 has Cartesian equation $\frac{x-1}{3} = \frac{y+2}{1} = \frac{z-1}{-2}$. [5]

Show that L_1 and L_2 are skew lines.

c. Find the Cartesian equation of the plane Π_1. [4]

d. (i) Find the value of k. [7]

(ii) Find the point of intersection P of the line L_3 and the plane Π_2.

Markscheme

a. direction vector $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or $\overrightarrow{\mathrm{BA}} = \begin{pmatrix} -1 \\ -3 \\ 5 \end{pmatrix}$ ***A1***

$\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or $\boldsymbol{r} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$ or equivalent ***A1***

Note: Do not award final ***A1*** unless '$\boldsymbol{r} = \mathrm{K}$' (or equivalent) seen.
Allow FT on direction vector for final ***A1***.

[2 marks]

b. both lines expressed in parametric form:

L_1:

$x = 1 + t$

$y = 3t$

$z = 4 - 5t$

L_2:

$x = 1 + 3s$

$y = -2 + s$ ***M1A1***

$z = -2s + 1$

Notes: Award ***M1*** for an attempt to convert L_2 from Cartesian to parametric form.
Award ***A1*** for correct parametric equations for L_1 and L_2.
Allow ***M1A1*** at this stage if same parameter is used in both lines.

attempt to solve simultaneously for *x* and *y*: ***M1***

$1 + t = 1 + 3s$

$3t = -2 + s$

$t = -\frac{3}{4},\ s = -\frac{1}{4}$ ***A1***

substituting both values back into z values respectively gives $z = \frac{31}{4}$

and $z = \frac{3}{2}$ so a contradiction ***R1***

therefore L_1 and L_1 are skew lines ***AG***

[5 marks]

c. finding the cross product:

$\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ ***(M1)***

$= -\boldsymbol{i} - 13\boldsymbol{j} - 8\boldsymbol{k}$ ***A1***

Note: Accept $\boldsymbol{i} + 13\boldsymbol{j} + 8\boldsymbol{k}$

$-1(0) - 13(1) - 8(-2) = 3$ ***(M1)***

$\Rightarrow -x - 13y - 8z = 3$ or equivalent ***A1***

[4 marks]

d.

(i) $(\cos\theta =)\frac{\begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{k^2+1+1} \times \sqrt{1+1}}$ ***M1***

Note: Award ***M1*** for an attempt to use angle between two vectors formula.

$\frac{\sqrt{3}}{2} = \frac{k+1}{\sqrt{2(k^2+2)}}$ ***A1***

obtaining the quadratic equation

$4(k+1)^2 = 6(k^2+2)$ ***M1***

$k^2 - 4k + 4 = 0$

$(k-2)^2 = 0$

$k = 2$ ***A1***

Note: Award ***M1A0M1A0*** if $\cos 60^\circ$ is used ($k = 0$ or $k = -4$).

(ii) $r = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

substituting into the equation of the plane Π_2:

$3 + 2\lambda + \lambda = 12$ ***M1***

$\lambda = 3$ ***A1***

point P has the coordinates:

(9, 3, –2) ***A1***

Notes: Accept $9\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ and $\begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$.

Do not allow FT if two values found for k.

[7 marks]

Examiners report

a. [N/A]

b. [N/A]

c. [N/A]

d. [N/A]

Given any two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that $|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2$.

Markscheme

METHOD 1

Use of $|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}|\sin\theta$ ***(M1)***

$|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ ***(A1)***

Note: Only one of the first two marks can be implied.

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2(1 - \cos^2\theta)$ ***A1***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ ***(A1)***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (|\boldsymbol{a}||\boldsymbol{b}|\cos\theta)^2$ ***(A1)***

Note: Only one of the above two ***A1*** marks can be implied.

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2$ ***A1***

Hence LHS = RHS ***AG*** ***N0***

[6 marks]

METHOD 2

Use of $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$ ***(M1)***

$|\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (|\boldsymbol{a}||\boldsymbol{b}|\cos\theta)^2$ ***(A1)***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ ***(A1)***

Note: Only one of the above two ***A1*** marks can be implied.

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2(1 - \cos^2\theta)$ ***A1***

$= |\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ ***A1***

$= |\boldsymbol{a} \times \boldsymbol{b}|^2$ ***A1***

Hence LHS = RHS ***AG*** ***N0***

Notes: Candidates who independently correctly simplify both sides and show that LHS = RHS should be awarded full marks.

If the candidate starts off with expression that they are trying to prove and concludes that $\sin^2\theta = (1 - \cos^2\theta)$ award ***M1A1A1A1A0A0***.

If the candidate uses two general 3D vectors and explicitly finds the expressions correctly award full marks. Use of 2D vectors gains a maximum of 2 marks.

If two specific vectors are used no marks are gained.

[6 marks]

Examiners report

Those candidates who chose to use the trigonometric version of Pythagoras' Theorem were generally successful, although a minority were unconvincing in their reasoning. Some candidates adopted a full component approach, but often seemed to lose track of what they were trying to prove. A few candidates used 2-dimensional vectors or specific rather than general vectors.

The points A and B are given by $\text{A}(0,\ 3,\ -6)$ and $\text{B}(6,\ -5,\ 11)$.

The plane *Π* is defined by the equation $4x - 3y + 2z = 20$.

a. Find a vector equation of the line *L* passing through the points A and B. [3]

b. Find the coordinates of the point of intersection of the line *L* with the plane *Π*. [3]

Markscheme

a. $\overrightarrow{\text{AB}} = \begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix}$ ***(A1)***

r = $\begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix}$ or ***r*** = $\begin{pmatrix} 6 \\ -5 \\ 11 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix}$ ***M1A1***

Note: Award ***M1A0*** if ***r*** = is not seen (or equivalent).

[3 marks]

b. substitute line *L* in $\Pi : 4(6\lambda) - 3(3 - 8\lambda) + 2(-6 + 17\lambda) = 20$ ***M1***

$82\lambda = 41$

$\lambda = \frac{1}{2}$ ***(A1)***

r = $\begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ -8 \\ 17 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ \frac{5}{2} \end{pmatrix}$

so coordinate is $\left(3,\ -1,\ \frac{5}{2}\right)$ ***A1***

Note: Accept coordinate expressed as position vector $\begin{pmatrix} 3 \\ -1 \\ \frac{5}{2} \end{pmatrix}$.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The three vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are given by

$$\boldsymbol{a} = \begin{pmatrix} 2y \\ -3x \\ 2x \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 4x \\ y \\ 3-x \end{pmatrix}, \boldsymbol{c} = \begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix} \text{ where } x, y \in \mathbb{R}.$$

(a) If $\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} = 0$, find the value of x and of y.

(b) Find the exact value of $|\boldsymbol{a} + 2\boldsymbol{b}|$.

Markscheme

(a) $2y + 8x = 4$ ***M1***

$-3x + 2y = -7$ ***A1***

$2x + 6 - 2x = 6$

Note: Award ***M1*** for attempt at components, ***A1*** for two correct equations.

No penalty for not checking the third equation.

solving : $x = 1, y = -2$ ***A1***

(b) $|\mathbf{a} + 2\boldsymbol{b}| = \left|\begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}\right|$

$= \left|\begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix}\right|$

$\Rightarrow |\boldsymbol{a} + 2\boldsymbol{b}| = \sqrt{4^2 + (-7)^2 + 6^2}$ ***(M1)***

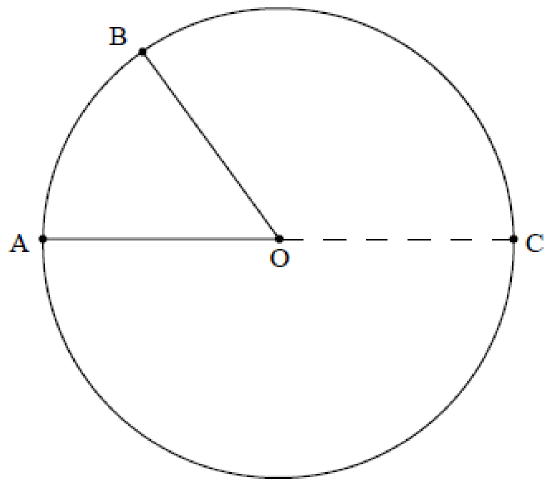
$= \sqrt{101}$ ***A1***

[5 marks]

Examiners report

The majority of candidates understood what was required in part (a) of this question and gained the correct answer. Most candidates were able to do part (b) but few realised that they did not have to calculate $|\boldsymbol{a} + 2\boldsymbol{b}|$ as this is $|\boldsymbol{c}|$. Many candidates lost time on this question.

The diagram below shows a circle with centre O. The points A, B, C lie on the circumference of the circle and [AC] is a diameter.



Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

a. Write down expressions for $\overrightarrow{AB}$ and $\overrightarrow{CB}$ in terms of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. [2]

b. Hence prove that angle $A\hat{B}C$ is a right angle. [3]

Markscheme

a. $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ ***A1***

$\overrightarrow{CB} = \boldsymbol{a} + \boldsymbol{b}$ ***A1***

[2 marks]

b. $\overrightarrow{AB} \cdot \overrightarrow{CB} = (\boldsymbol{b} - \boldsymbol{a}) \cdot (\boldsymbol{b} + \boldsymbol{a})$ ***M1***

$= |\mathbf{b}|^2 - |\mathbf{a}|^2$ ***A1***

$= 0$ since $|\boldsymbol{b}| = |\boldsymbol{a}|$ ***R1***

Note: Only award the ***A1*** and ***R1*** if working indicates that they understand that they are working with vectors.

so $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CB}$ *i.e.* $A\hat{B}C$ is a right angle ***AG***

[3 marks]

Examiners report

a. This question was poorly done with most candidates having difficulties in using appropriate notation which made unclear the distinction between scalars and vectors. A few candidates scored at least one of the marks in (a) but most candidates had problems in setting up the proof required in (b) with many using a circular argument which resulted in a very poor performance in this part.

b. This question was poorly done with most candidates having difficulties in using appropriate notation which made unclear the distinction between scalars and vectors. A few candidates scored at least one of the marks in (a) but most candidates had problems in setting up the proof required in (b) with many using a circular argument which resulted in a very poor performance in this part.

Two planes have equations

$$\Pi_1 : \ 4x + y + z = 8 \text{ and } \Pi_2 : \ 4x + 3y - z = 0$$

Let L be the line of intersection of the two planes.

B is the point on Π_1 with coordinates $(a,\ b,\ 1)$.

The point P lies on L and $\mathrm{A\hat{B}P} = 45^\circ$.

a. Find the cosine of the angle between the two planes in the form $\sqrt{\frac{p}{q}}$ where $p,\ q \in \mathbb{Z}$. [4]

b. (i) Show that L has direction $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$. [6]

(ii) Show that the point $\mathrm{A}\ (1,\ 0,\ 4)$ lies on both planes.

(iii) Write down a vector equation of L.

c. Given the vector $\overrightarrow{\mathrm{AB}}$ is perpendicular to L find the value of a and the value of b. [5]

d. Show that $\mathrm{AB} = 3\sqrt{2}$. [1]

e. Find the coordinates of the two possible positions of P. [5]

Markscheme

a. **Note:** Throughout the question condone vectors written horizontally.

angle between planes is equal to the angles between the normal to the planes ***(M1)***

$\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = 18$ ***(A1)***

let θ be the angle between the normal to the planes

$\cos\theta = \frac{18}{\sqrt{18}\sqrt{26}} = \sqrt{\frac{18}{26}} \left(\text{or equivalent, for example } \sqrt{\frac{324}{468}} \text{ or } \sqrt{\frac{9}{13}}\right)$ ***M1A1***

[4 marks]

b. **Note:** Throughout the question condone vectors written horizontally.

(i) **METHOD 1**

$$\begin{pmatrix}4\\1\\1\end{pmatrix}\times\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}-4\\8\\8\end{pmatrix}$$ ***M1A1***

which is a multiple of $\begin{pmatrix}-1\\2\\2\end{pmatrix}$ ***R1AG***

Note: Allow any equivalent wording or $\begin{pmatrix}-4\\8\\8\end{pmatrix}=4\begin{pmatrix}-1\\2\\2\end{pmatrix}$, do not allow $\begin{pmatrix}-4\\8\\8\end{pmatrix}=\begin{pmatrix}-1\\2\\2\end{pmatrix}$.

METHOD 2

let $z=t$ (or equivalent)

solve simultaneously to get ***M1***

$y=t-4,\ x=3-0.5t$ ***A1***

hence direction vector is $\begin{pmatrix}-0.5\\1\\1\end{pmatrix}$

which is a multiple of $\begin{pmatrix}-1\\2\\2\end{pmatrix}$ ***R1AG***

METHOD 3

$\begin{pmatrix}4\\1\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\2\end{pmatrix}=-4+2+2=0$ ***M1A1***

$\begin{pmatrix}4\\3\\-1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\2\end{pmatrix}=-4+6-2=0$ ***A1***

Note: If only one scalar product is found award ***M0A0A0***.

(ii) $\Pi_1:\ 4+0+4=8$ and $\Pi_2:\ 4+0-4=0$ ***R1***

(iii) $\boldsymbol{r}=\begin{pmatrix}1\\0\\4\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\2\end{pmatrix}$ ***A1A1***

Note: ***A1*** for "$\boldsymbol{r}=$" and a correct point on the line, ***A1*** for a parameter and a correct direction vector.

[6 marks]

c. **Note:** Throughout the question condone vectors written horizontally.

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}a\\b\\1\end{pmatrix}-\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}a-1\\b\\-3\end{pmatrix}$ ***(A1)***

$\begin{pmatrix}a-1\\b\\-3\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\2\end{pmatrix}=0$ ***M1***

Note: Award ***M0*** for $\begin{pmatrix}a\\b\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\2\end{pmatrix}=0$.

$-a+1+2b-6=0\Rightarrow a-2b=-5$ ***A1*** lies on Π_1 so $4a + b + 1 = 8 \Rightarrow 4a + b = 7$ ***M1***

$a = 1,\ b = 3$ ***A1***

[5 marks]

d. **Note:** Throughout the question condone vectors written horizontally.

$\mathrm{AB} = \sqrt{0^2 + 3^2 + (-3)^2} = 3\sqrt{2}$ ***M1AG***

[1 mark]

e. **Note:** Throughout the question condone vectors written horizontally.

METHOD 1

$\left|\overrightarrow{\mathrm{AB}}\right| = \left|\overrightarrow{\mathrm{AP}}\right| = 3\sqrt{2}$ ***(M1)***

$\overrightarrow{\mathrm{AP}} = t\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ ***(A1)***

$|3t| = 3\sqrt{2} \Rightarrow t = \pm\sqrt{2}$ ***(M1)A1***

$\mathrm{P}\left(1 - \sqrt{2},\ 2\sqrt{2},\ 4 + 2\sqrt{2}\right)$ and $\left(1 + \sqrt{2},\ -2\sqrt{2},\ 4 - 2\sqrt{2}\right)$ ***A1***

[5 marks]

METHOD 2

let P have coordinates $(1 - \lambda,\ 2\lambda,\ 4 + 2\lambda)$ ***M1***

$\overrightarrow{\mathrm{BA}} = \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix}, \overrightarrow{\mathrm{BP}} = \begin{pmatrix} -\lambda \\ 2\lambda - 3 \\ 3 + 2\lambda \end{pmatrix}$ ***A1***

$\cos 45^\circ = \frac{\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BP}}}{|\mathrm{BA}||\mathrm{BP}|}$ ***M1***

Note: Award ***M1*** even if AB rather than BA is used in the scalar product.

$\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BP}} = 18$

$\frac{1}{\sqrt{2}} = \frac{18}{\sqrt{18}\sqrt{9\lambda^2 + 18}}$

$\lambda = \pm\sqrt{2}$ ***A1***

$\mathrm{P}\left(1 - \sqrt{2},\ 2\sqrt{2},\ 4 + 2\sqrt{2}\right)$ and $\left(1 + \sqrt{2},\ -2\sqrt{2},\ 4 - 2\sqrt{2}\right)$ ***A1***

Note: Accept answers given as position vectors.

[5 marks]

Examiners report

a. This was successfully done, though some candidates lost marks unnecessarily by not giving the answer in the form requested in the question.

b. (b)(i) A variety of techniques were successfully used here. A common error was not to justify why the vector obtained from the vector product was in the same direction as the one given in the question.

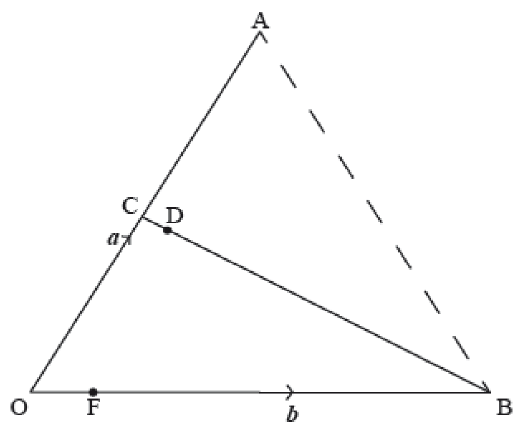
(b)(ii) and (iii) These were well done, though too many candidates still lose a mark unnecessarily by writing the vector equation of a line as $l =$ rather than $r =$.

c. Weaker candidates found the rest of this question more difficult. Though most obtained one equation for a and b they did not take note of the fact that it was also on the given plane, which gave the second equation.

d. This was done successfully by the majority of candidates. Candidates need to be aware that the notation AB means the length of the line segment joining the points A and B (as in the course guide).

e. This proved to be a difficult question for most candidates. Those who were successful were equally split between the two approaches given in the markscheme.

In the following diagram, $\overrightarrow{\mathrm{OA}}$ = ***a***, $\overrightarrow{\mathrm{OB}}$ = ***b***. C is the midpoint of [OA] and $\overrightarrow{\mathrm{OF}} = \frac{1}{6}\overrightarrow{\mathrm{FB}}$.



It is given also that $\overrightarrow{\mathrm{AD}} = \lambda\overrightarrow{\mathrm{AF}}$ and $\overrightarrow{\mathrm{CD}} = \mu\overrightarrow{\mathrm{CB}}$, where $\lambda,\ \mu \in \mathbb{R}$.

a.i. Find, in terms of ***a*** and ***b*** $\overrightarrow{\mathrm{OF}}$. [1]

a.ii. Find, in terms of ***a*** and ***b*** $\overrightarrow{\mathrm{AF}}$. [2]

b.i. Find an expression for $\overrightarrow{\mathrm{OD}}$ in terms of ***a***, ***b*** and λ; [2]

b.ii. Find an expression for $\overrightarrow{\mathrm{OD}}$ in terms of ***a***, ***b*** and μ. [2]

c. Show that $\mu = \frac{1}{13}$, and find the value of λ. [4]

d. Deduce an expression for $\overrightarrow{\mathrm{CD}}$ in terms of ***a*** and ***b*** only. [2]

e. Given that area $\Delta\mathrm{OAB} = k(\text{area } \Delta\mathrm{CAD})$, find the value of k. [5]

Markscheme

a.i. $\overrightarrow{\mathrm{OF}} = \frac{1}{7}$***b*** ***A1***

[1 mark]

a.ii. $\overrightarrow{\mathrm{AF}} = \overrightarrow{\mathrm{OF}} - \overrightarrow{\mathrm{OA}}$ ***(M1)***

$= \frac{1}{7}\boldsymbol{b} - \boldsymbol{a}$ ***A1***

[2 marks]

b.i. $\overrightarrow{\mathrm{OD}} = \boldsymbol{a} + \lambda\left(\frac{1}{7}b - a\right)\ \left(= (1-\lambda)a + \frac{\lambda}{7}b\right)$ ***M1A1***

[2 marks]

b.ii. $\overrightarrow{\mathrm{OD}} = \frac{1}{2}\boldsymbol{a} + \mu\left(-\frac{1}{2}a + b\right)\ \left(= \left(\frac{1}{2} - \frac{\mu}{2}\right)a + \mu b\right)$ ***M1A1***

[2 marks]

c. equating coefficients: ***M1***

$\frac{\lambda}{7} = \mu,\ 1 - \lambda = \frac{1-\mu}{2}$ ***A1***

solving simultaneously: ***M1***

$\lambda = \frac{7}{13},\ \mu = \frac{1}{13}$ ***A1AG***

[4 marks]

d. $\overrightarrow{\mathrm{CD}} = \frac{1}{13}\overrightarrow{\mathrm{CB}}$

$= \frac{1}{13}\left(b - \frac{1}{2}a\right)\ \left(= -\frac{1}{26}a + \frac{1}{13}b\right)$ ***M1A1***

[2 marks]

e. **METHOD 1**

$\text{area } \Delta\mathrm{ACD} = \frac{1}{2}\mathrm{CD} \times \mathrm{AC} \times \sin \mathrm{A\hat{C}B}$ ***(M1)***

$\text{area } \Delta\mathrm{ACB} = \frac{1}{2}\mathrm{CB} \times \mathrm{AC} \times \sin \mathrm{A\hat{C}B}$ ***(M1)***

$\text{ratio } \frac{\text{area } \Delta\mathrm{ACD}}{\text{area } \Delta\mathrm{ACB}} = \frac{\mathrm{CD}}{\mathrm{CB}} = \frac{1}{13}$ ***A1***

$k = \frac{\text{area } \Delta\mathrm{OAB}}{\text{area } \Delta\mathrm{CAD}} = \frac{13}{\text{area } \Delta\mathrm{CAB}} \times \text{area } \Delta\mathrm{OAB}$ ***(M1)***

$= 13 \times 2 = 26$ ***A1***

METHOD 2

$\text{area } \Delta\mathrm{OAB} = \frac{1}{2}|a \times b|$ ***A1***

$\text{area } \Delta\mathrm{CAD} = \frac{1}{2}\left|\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CD}}\right|$ or $\frac{1}{2}\left|\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{AD}}\right|$ ***M1***

$= \frac{1}{2}\left|\frac{1}{2}a \times \left(-\frac{1}{26}a + \frac{1}{13}b\right)\right|$

$= \frac{1}{2}\left|\frac{1}{2}a \times \left(-\frac{1}{26}a\right) + \frac{1}{2}a \times \frac{1}{13}b\right|$ ***(M1)***

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{13}|a \times b|\ \left(= \frac{1}{52}|a \times b|\right)$ ***A1***

$\text{area } \Delta\mathrm{OAB} = k(\text{area } \Delta\mathrm{CAD})$

$\frac{1}{2}|a \times b| = k\frac{1}{52}|a \times b|$

$k = 26$ ***A1***

[5 marks]

Examiners report

[N/A]

a.ii. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The vertices of a triangle ABC have coordinates given by A(−1, 2, 3), B(4, 1, 1) and C(3, −2, 2).

a. (i) Find the lengths of the sides of the triangle. [6]

(ii) Find $\cos \mathrm{B\hat{A}C}$.

b. (i) Show that $\overrightarrow{\mathrm{BC}} \times \overrightarrow{\mathrm{CA}} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$. [5]

(ii) Hence, show that the area of the triangle ABC is $\frac{1}{2}\sqrt{314}$.

c. Find the Cartesian equation of the plane containing the triangle ABC. [3]

d. Find a vector equation of (AB). [2]

e. The point D on (AB) is such that $\overrightarrow{\mathrm{OD}}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$ where O is the origin. [5]

(i) Find the coordinates of D.

(ii) Show that D does not lie between A and B.

Markscheme

a. (i) $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}} = 5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ (or in column vector form) ***(A1)***

Note: Award ***A1*** if any one of the vectors, or its negative, representing the sides of the triangle is seen.

$\overrightarrow{\mathrm{AB}} = |5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}| = \sqrt{30}$
$\overrightarrow{\mathrm{BC}} = |-\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}| = \sqrt{11}$
$\overrightarrow{\mathrm{CA}} = |-4\boldsymbol{i} + 4\boldsymbol{j} + \boldsymbol{k}| = \sqrt{33}$ ***A2***

Note: Award ***A1*** for two correct and ***A0*** for one correct.

(ii) **METHOD 1**

$\cos \mathrm{BAC} = \frac{20+4+2}{\sqrt{30}\sqrt{33}}$ ***M1A1***

Note: Award ***M1*** for an attempt at the use of the scalar product for two vectors representing the sides AB and AC, or their negatives, ***A1*** for the correct computation using their vectors.

$= \frac{26}{\sqrt{990}} \left(= \frac{26}{3\sqrt{110}}\right)$ ***A1***

Note: Candidates who use the modulus need to justify it – the angle is not stated in the question to be acute.

METHOD 2

using the cosine rule

$\cos \mathrm{BAC} = \frac{30+33-11}{2\sqrt{30}\sqrt{33}}$ ***M1A1***

$= \frac{26}{\sqrt{990}} \left(= \frac{26}{3\sqrt{110}}\right)$ ***A1***

[6 marks]

b. $\overrightarrow{\mathrm{BC}} \times \overrightarrow{\mathrm{CA}} = \begin{vmatrix} i & j & k \\ -1 & -3 & 1 \\ -4 & 4 & 1 \end{vmatrix}$ ***A1***

$= ((-3) \times 1 - 1 \times 4)\boldsymbol{i} + (1 \times (-4) - (-1) \times 1)\boldsymbol{j} + ((-1) \times 4 - (-3) \times (-4))\boldsymbol{k}$ ***M1A1***

$= -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$ ***AG***

(ii) the area of $\Delta\mathrm{ABC} = \frac{1}{2}\left|\overrightarrow{\mathrm{BC}} \times \overrightarrow{\mathrm{CA}}\right|$ ***(M1)***

$\frac{1}{2}\sqrt{(-7)^2 + (-3)^2 + (-16)^2}$ ***A1***

$= \frac{1}{2}\sqrt{314}$ ***AG***

[5 marks]

c. attempt at the use of "$(\boldsymbol{r} - \boldsymbol{a})\cdot\boldsymbol{n} = 0$" ***(M1)***

using $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, $\boldsymbol{a} = \overrightarrow{\mathrm{OA}}$ and $\boldsymbol{n} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$ ***(A1)***

$7x + 3y + 16z = 47$ ***A1***

Note: Candidates who adopt a 2-parameter approach should be awarded, ***A1*** for correct 2-parameter equations for x, y and z; ***M1*** for a serious attempt at elimination of the parameters; ***A1*** for the final Cartesian equation.

[3 marks]

d. $\boldsymbol{r} = \overrightarrow{\mathrm{OA}} + t\overrightarrow{\mathrm{AB}}$ (or equivalent) ***M1***

$\boldsymbol{r} = (-\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + t\,(5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k})$ ***A1***

Note: Award ***M1A0*** if "$\boldsymbol{r} =$" is missing.

Note: Accept forms of the equation starting with B or with the direction reversed.

[2 marks]

e. (i) $\overrightarrow{\mathrm{OD}} = (-\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + t(5\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k})$

statement that $\overrightarrow{\mathrm{OD}} \cdot \overrightarrow{\mathrm{BC}} = 0$ ***(M1)***

$\begin{pmatrix} -1+5t \\ 2-t \\ 3-2t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} = 0$ ***A1***

$-2 - 4t = 0$ or $t = -\frac{1}{2}$ ***A1***

coordinates of D are $\left(-\frac{7}{2}, \frac{5}{2}, 4\right)$ ***A1***

Note: Different forms of $\overrightarrow{\mathrm{OD}}$ give different values of t, but the same final answer.

(ii) $t < 0 \Rightarrow$ D is not between A and B ***R1***

[5 marks]

Examiners report

a. Many candidates confidently tackled most of this many-part question. Part (b)(i) As the answer was given, candidates were required to show they really knew how to work out a vector product in detail, not just by writing down a 3x3 determinant and then the final answer. Part (d) A few candidates failed to realise that the equation of a line is an equation not simply an expression. Part (e) A significant number of candidates did not realise that they could use their result for part (d).

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A line L has equation $\frac{x-2}{p} = \frac{y-q}{2} = z - 1$ where $p,\ q \in \mathbb{R}$.

A plane Π has equation $x + y + 3z = 9$.

Consider the different case where the acute angle between L and Π is θ

where $\theta = \arcsin\left(\frac{1}{\sqrt{11}}\right)$.

a. Show that L is not perpendicular to Π. [3]

b. Given that L lies in the plane Π, find the value of p and the value of q. [4]

c. (i) Show that $p = -2$. [11]

(ii) If L intersects Π at $z = -1$, find the value of q.

Markscheme

a. **EITHER**

n $= \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$ and ***d*** $= \begin{pmatrix} p \\ 2 \\ 1 \end{pmatrix}$ ***A1A1***

and ***n*** $\neq k$***d*** ***R1***

OR

n $\times$ ***d*** $= \begin{pmatrix} -5 \\ 3p-1 \\ 2-p \end{pmatrix}$ ***M1A1***

the vector product is non-zero for $p \in \mathbb{R}$ ***R1***

THEN

L is not perpendicular to Π ***AG***

[3 marks]

b. **METHOD 1**

$(2 + p\lambda) + (q + 2\lambda) + 3(1 + \lambda) = 9$ ***M1***

$(q + 5) + (p + 5)\lambda = 9$ ***(A1)***

$p = -5$ and $q = 4$ ***A1A1***

METHOD 2

direction vector of line is perpendicular to plane, so

$\begin{pmatrix} p \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 0$ ***M1***

$p = -5$ ***A1***

$(2,\ q,\ 1)$ is common to both L and Π

either $\begin{pmatrix} 2 \\ q \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} = 9$ or by substituting into $x + y + 3z = 9$ ***M1***

$q = 4$ ***A1***

[4 marks]

c. (i) **METHOD 1**

α is the acute angle between ***n*** and *L*

if $\sin\theta = \frac{1}{\sqrt{11}}$ then $\cos\alpha = \frac{1}{\sqrt{11}}$ ***(M1)(A1)***

attempting to use $\cos\alpha = \frac{n \bullet d}{|n||d|}$ or $\sin\alpha = \frac{n \bullet d}{|n||d|}$ ***M1***

$\frac{p+5}{\sqrt{11} \times \sqrt{p^2+5}} = \frac{1}{\sqrt{11}}$ ***A1A1***

$(p + 5)^2 = p^2 + 5$ ***M1***

$10p = -20$ (or equivalent) ***A1***

$p = -2$ ***AG***

METHOD 2

α is the angle between ***n*** and *L*

if $\sin\theta = \frac{1}{\sqrt{11}}$ then $\sin\alpha = \frac{\sqrt{10}}{\sqrt{11}}$ ***(M1)A1***

attempting to use $\sin\alpha = \frac{|n\times d|}{|n||d|}$ ***M1***

$\frac{\sqrt{(-5)^2+(3p-1)^2+(2-p)^2}}{\sqrt{11}\times\sqrt{p^2+5}} = \frac{\sqrt{10}}{\sqrt{11}}$ ***A1A1***

$p^2 - p + 3 = p^2 + 5$ ***M1***

$-p + 3 = 5$ (or equivalent) ***A1***

$p = -2$ ***AG***

(ii) $p = -2$ and $z = -1 \Rightarrow \frac{x-2}{-2} = \frac{y-q}{2} = -2$ ***(A1)***

$x = 6$ and $y = q - 4$ ***(A1)***

this satisfies Π so $6 + q - 4 - 3 = 9$ ***M1***

$q = 10$ ***A1***

[11 marks]

Examiners report

a. Parts (a) and (b) were often well done, though a small number of candidates were clearly puzzled when trying to demonstrate $\begin{pmatrix}1\\1\\3\end{pmatrix} \neq k\begin{pmatrix}p\\2\\1\end{pmatrix}$, with some scripts seen involving needlessly convoluted arguments.

b. Parts (a) and (b) were often well done, though a small number of candidates were clearly puzzled when trying to demonstrate $\begin{pmatrix}1\\1\\3\end{pmatrix} \neq k\begin{pmatrix}p\\2\\1\end{pmatrix}$, with some scripts seen involving needlessly convoluted arguments.

c. Part (c) often proved problematic, as some candidates unsurprisingly used the sine (or cosine) of an incorrect angle, and few consequent marks were then available. Some good clear solutions were seen, occasionally complete with diagrams in the cases of the thoughtful candidates who were able to 'work through' the question rather than just apply a standard vector result.

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a. For non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that [8]

(i) if $|\boldsymbol{a} - \boldsymbol{b}| = |\boldsymbol{a} + \boldsymbol{b}|$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular;

(ii) $|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a}\cdot\boldsymbol{b})^2$.

b. The points A, B and C have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. [7]

(i) Show that the area of triangle ABC is $\frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b} + \boldsymbol{b}\times\boldsymbol{c} + \boldsymbol{c}\times\boldsymbol{a}|$.

(ii) Hence, show that the shortest distance from B to AC is

$$\frac{|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|}{|\boldsymbol{c}-\boldsymbol{a}|}.$$

Markscheme

a. (i) $|\boldsymbol{a}-\boldsymbol{b}|=|\boldsymbol{a}+\boldsymbol{b}|$

$\Rightarrow (\boldsymbol{a}-\boldsymbol{b})\cdot(\boldsymbol{a}-\boldsymbol{b})=(\boldsymbol{a}+\boldsymbol{b})\cdot(\boldsymbol{a}+\boldsymbol{b})$ ***(M1)***

$\Rightarrow |\boldsymbol{a}|^2-2\boldsymbol{a}\cdot\boldsymbol{b}+|\boldsymbol{b}|^2=|\boldsymbol{a}|^2+2\boldsymbol{a}\cdot\boldsymbol{b}+|\boldsymbol{b}|^2$ ***A1***

$\Rightarrow 4\boldsymbol{a}\cdot\boldsymbol{b}=0\Rightarrow \boldsymbol{a}\cdot\boldsymbol{b}=0$ ***A1***

therefore $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular ***R1***

Note: Allow use of 2-d components.

Note: Do not condone sloppy vector notation, so we must see something to the effect that $|\boldsymbol{c}|^2=\boldsymbol{c}\cdot\boldsymbol{c}$ is clearly being used for the ***M1***.

Note: Allow a correct geometric argument, for example that the diagonals of a parallelogram have the same length only if it is a rectangle.

(ii) $|\boldsymbol{a}\times\boldsymbol{b}|^2=(|\boldsymbol{a}|\,|\boldsymbol{b}|\sin\theta)^2=|\boldsymbol{a}|^2|\boldsymbol{b}|^2\sin^2\theta$ ***M1A1***

$|\boldsymbol{a}|^2|\boldsymbol{b}|^2-(\boldsymbol{a}\cdot\boldsymbol{b})^2=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-|\boldsymbol{a}|^2|\boldsymbol{b}|^2\cos^2\theta$ ***M1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2\left(1-\cos^2\theta\right)$ ***A1***

$=|\boldsymbol{a}|^2|\boldsymbol{b}|^2\left(\sin^2\theta\right)$

$\Rightarrow |\boldsymbol{a}\times\boldsymbol{b}|^2=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-(\boldsymbol{a}\cdot\boldsymbol{b})^2$ ***AG***

[8 marks]

b. (i) area of triangle $=\frac{1}{2}\left|\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}\right|$ ***(M1)***

$=\frac{1}{2}|(\boldsymbol{b}-\boldsymbol{a})\times(\boldsymbol{c}-\boldsymbol{a})|$ ***A1***

$=\frac{1}{2}|\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{b}\times-\boldsymbol{a}+-\boldsymbol{a}\times\boldsymbol{c}+-\boldsymbol{a}\times-\boldsymbol{a}|$ ***A1***

$\boldsymbol{b}\times-\boldsymbol{a}=\boldsymbol{a}\times\boldsymbol{b};\ \boldsymbol{c}\times\boldsymbol{a}=-\boldsymbol{a}\times\boldsymbol{c};\ -\boldsymbol{a}\times-\boldsymbol{a}=0$ ***M1***

hence, area of triangle is $\frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|$ ***AG***

(ii) D is the foot of the perpendicular from B to AC

area of triangle ABC $=\frac{1}{2}\left|\overrightarrow{\mathrm{AC}}\right|\left|\overrightarrow{\mathrm{BD}}\right|$ ***A1***

therefore

$\frac{1}{2}\left|\overrightarrow{\mathrm{AC}}\right|\left|\overrightarrow{\mathrm{BD}}\right|=\frac{1}{2}\left|\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}\right|$ ***M1***

hence, $\left|\overrightarrow{\mathrm{BD}}\right|=\frac{\left|\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}\right|}{\left|\overrightarrow{\mathrm{AC}}\right|}$ ***A1***

$=\frac{|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|}{|\boldsymbol{c}-\boldsymbol{a}|}$ ***AG***

[7 marks]

Examiners report

a. (i) The majority of candidates were very sloppy in their use of vector notation. Some candidates used Cartesian coordinates, which was acceptable. Part (ii) was well done.

b. Part (i) was usually well started, but not completed satisfactorily. Many candidates understood the geometry involved in this part.

Consider the points A(1, −1, 4), B (2, − 2, 5) and O(0, 0, 0).

(a) Calculate the cosine of the angle between $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{AB}}$.

(b) Find a vector equation of the line L_1 which passes through A and B.

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$, where $t \in \mathbb{R}$.

(c) Show that the lines L_1 and L_2 intersect and find the coordinates of their point of intersection.

(d) Find the Cartesian equation of the plane which contains both the line L_2 and the point A.

Markscheme

(a) Use of $\cos\theta = \frac{\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{AB}}}{\left|\overrightarrow{\mathrm{OA}}\right|\left|\overrightarrow{\mathrm{AB}}\right|}$ ***(M1)***

$\overrightarrow{\mathrm{AB}} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ ***A1***

$\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{3}$ **and** $\left|\overrightarrow{\mathrm{OA}}\right| = 3\sqrt{2}$ ***A1***

$\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{AB}} = 6$ ***A1***

substituting gives $\cos\theta = \frac{2}{\sqrt{6}} \left(= \frac{\sqrt{6}}{3}\right)$ or equivalent ***M1 N1***

[5 marks]

(b) $L_1 : \boldsymbol{r} = \overrightarrow{\mathrm{OA}} + s\overrightarrow{\mathrm{AB}}$ or equivalent ***(M1)***

$L_1 : \boldsymbol{r} = \boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k} + s(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$ or equivalent ***A1***

Note: Award ***(M1)A0*** for omitting " $\boldsymbol{r} =$ " in the final answer.

[2 marks]

(c) Equating components and forming equations involving s and t ***(M1)***

$1 + s = 2 + 2t$, $-1 - s = 4 + t$, $4 + s = 7 + 3t$

Having two of the above three equations ***A1A1***

Attempting to solve for s or t ***(M1)***

Finding either $s = -3$ or $t = -2$ ***A1***

Explicitly showing that these values satisfy the third equation ***R1***

Point of intersection is (−2, 2, 1) ***A1 N1***

Note: Position vector is not acceptable for final ***A1***.

[7 marks]

(d) **METHOD 1**

$$r = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix} \quad \textit{(A1)}$$

$x = 1 + 2\lambda - 3\mu$, $y = -1 + \lambda + 3\mu$ and $z = 4 + 3\lambda - 3\mu$ ***M1A1***

Elimination of the parameters ***M1***

$x + y = 3\lambda$ so $4(x + y) = 12\lambda$ and $y + z = 4\lambda + 3$ so $3(y + z) = 12\lambda + 9$

$3(y + z) = 4(x + y) + 9$ ***A1***

Cartesian equation of plane is $4x + y - 3z = -9$ (or equivalent) ***A1*** ***N1***

[6 marks]

METHOD 2

EITHER

The point (2, 4, 7) lies on the plane.

The vector joining (2, 4, 7) and (1, −1, 4) and $2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ are parallel to the plane. So they are perpendicular to the normal to the plane.

$(\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) - (2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k}) = -\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$ ***(A1)***

$$n = \begin{vmatrix} i & j & k \\ -1 & -5 & -3 \\ 2 & 1 & 3 \end{vmatrix}$$ ***M1***

$= -12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k}$ or equivalent parallel vector ***A1***

OR

L_1 and L_2 intersect at D(−2, 2, 1)

$$n = \begin{vmatrix} i & j & k \\ 2 & 1 & -3 \\ -3 & 3 & -3 \end{vmatrix}$$ ***M1***

$= -12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k}$ or equivalent parallel vector ***A1***

THEN

$\boldsymbol{r}\cdot\boldsymbol{n} = (\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})\cdot(-12\boldsymbol{i} - 3\boldsymbol{j} + 9\boldsymbol{k})$ ***M1***

$= 27$ ***A1***

Cartesian equation of plane is $4x + y - 3z = -9$ (or equivalent) ***A1*** ***N1***

[6 marks]

Total [20 marks]

Examiners report

Most candidates scored reasonably well on this question. The most common errors were: Using ***OB*** rather than ***AB*** in (a); omitting the $r =$ in (b); failure to check that the values of the two parameters satisfied the third equation in (c); the use of an incorrect vector in (d). Even when (d) was correctly answered, there was usually little evidence of why a specific vector had been used.

ABCD is a parallelogram, where $\overrightarrow{AB} = -\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\overrightarrow{AD} = 4\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$.

a. Find the area of the parallelogram ABCD. [3]

b. By using a suitable scalar product of two vectors, determine whether $\hat{ABC}$ is acute or obtuse. [4]

Markscheme

a. $\overrightarrow{AB} \times \overrightarrow{AD} = -\boldsymbol{i} + 10\boldsymbol{j} - 7\boldsymbol{k}$ ***M1A1***

$$\text{area} = \left|\overrightarrow{AB} \times \overrightarrow{AD}\right| = \sqrt{1^2 + 10^2 + 7^2}$$

$$= 5\sqrt{6}\left(\sqrt{150}\right)$$ ***A1***

[3 marks]

b. **METHOD 1**

$\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AD}} = -4 - 2 - 6$ ***M1A1***

$= -12$

considering the sign of the answer

$\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AD}} < 0$, therefore angle $\mathrm{D\hat{A}B}$ is obtuse ***M1***

(as it is a parallelogram), $\mathrm{A\hat{B}C}$ is acute ***A1***

[4 marks]

METHOD 2

$\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BC}} = +4 + 2 + 6$ ***M1A1***

$= 12$ considering the sign of the answer ***M1***

$\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{BC}} > 0 \Rightarrow \mathrm{A\hat{B}C}$ is acute ***A1***

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

The points A, B, C have position vectors $\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $3\boldsymbol{i} + \boldsymbol{k}$ respectively and lie in the plane π .

(a) Find

(i) the area of the triangle ABC;

(ii) the shortest distance from C to the line AB;

(iii) the cartesian equation of the plane π .

The line L passes through the origin and is normal to the plane π , it intersects π at the

point D.

(b) Find

(i) the coordinates of the point D;

(ii) the distance of π from the origin.

Markscheme

(a) (i) **METHOD 1**

$\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ ***(A1)***

$\overrightarrow{\mathrm{AC}} = \boldsymbol{c} - \boldsymbol{a} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ ***(A1)***

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix}$ ***M1***

$= \boldsymbol{i}(-1+1) - \boldsymbol{j}(0-2) + \boldsymbol{k}\,(0-2)$ ***(A1)***

$= 2\boldsymbol{j} - 2\boldsymbol{k}$ ***A1***

Area of triangle ABC}} $= \frac{1}{2}|2\mathbf{j} - 2\mathbf{k}| = \frac{1}{2}\sqrt{8} \;\; (=\sqrt{2})$ sq. units ***M1A1***

Note: Allow ***FT*** on final ***A1***.

METHOD 2

$|\mathrm{AB}| = \sqrt{2}, \;\; |\mathrm{BC}| = \sqrt{12}, \;\; |\mathrm{AC}| = \sqrt{6}$ ***A1A1A1***

Using cosine rule, e.g. on $\hat{C}$ ***M1***

$\cos C = \frac{6+12-2}{2\sqrt{72}} = \frac{2\sqrt{2}}{3}$ ***A1***

$\therefore$ Area $\Delta\mathrm{ABC} = \frac{1}{2}ab\sin C$ ***M1***

$= \frac{1}{2}\sqrt{12}\sqrt{6}\sin\left(\arccos\frac{2\sqrt{2}}{3}\right)$

$= 3\sqrt{2}\sin\left(\arccos\frac{2\sqrt{2}}{3}\right) \;\; \left(=\sqrt{2}\right)$ ***A1***

Note: Allow ***FT*** on final ***A1***.

(ii) $\mathrm{AB} = \sqrt{2}$ ***A1***

$\sqrt{2} = \frac{1}{2}\mathrm{AB} \times h = \frac{1}{2}\sqrt{2} \times h$, h equals the shortest distance ***(M1)***

$\Rightarrow h = 2$ ***A1***

(iii) **METHOD 1**

(\pi \) has form $r \cdot \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = d$ ***(M1)***

Since (1, 1, 2) is on the plane

$d = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = 2 - 4 = -2$ ***M1A1***

Hence $r \cdot \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} = -2$

$2y - 2z = -2$ (or $y - z = -1$) ***A1***

METHOD 2

$r = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ ***(M1)***

$x = 1 + 2\mu$ (i)

$y = 1 + \lambda - \mu$ (ii)

$z = 2 + \lambda - \mu$ (iii) ***A1***

Note: Award ***A1*** for all three correct, ***A0*** otherwise.

From (i) $\mu = \frac{x-1}{2}$

substitute in (ii) $y = 1 + \lambda - \left(\frac{x-1}{2}\right)$

$\Rightarrow \lambda = y - 1 + \left(\frac{x-1}{2}\right)$

substitute λ and μ in (iii) ***M1***

$\Rightarrow z = 2 + y - 1 + \left(\frac{x-1}{2}\right) - \left(\frac{x-1}{2}\right)$

$\Rightarrow y - z = -1$ ***A1***

[14 marks]

(b) (i) The equation of OD is

$r = \lambda \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}, \left(\text{or } r = \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right)$ ***M1***

This meets π where

$2\lambda + 2\lambda = -1$ ***(M1)***

$\lambda = -\frac{1}{4}$ ***A1***

Coordinates of D are $\left(0, -\frac{1}{2}, \frac{1}{2}\right)$ ***A1***

(ii) $\left|\overrightarrow{\text{OD}}\right| = \sqrt{0 + \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$ ***(M1)A1***

[6 marks]

Total [20 marks]

Examiners report

It was disappointing to see that a number of candidates did not appear to be well prepared for this question and made no progress at all. There were a number of schools where no candidate made any appreciable progress with the question. A good number of students, however, were successful with part (a) (i). A good number of candidates were also successful with part a (ii) but few realised that the shortest distance was the height of the triangle. Candidates used a variety of methods to answer (a) (iii) but again a reasonable number of correct answers were seen. Candidates also had a reasonable degree of success with part (b), with a respectable number of correct answers seen.

Consider the points A(1, 2, 3), B(1, 0, 5) and C(2, −1, 4).

a. Find $\overrightarrow{\text{AB}} \times \overrightarrow{\text{AC}}$. [4]

b. Hence find the area of the triangle ABC. [2]

Markscheme

a. $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ ***A1A1***

Note: Award the above marks if the components are seen in the line below.

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & -2 & 2 \\ 1 & -3 & 1 \end{vmatrix} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$ ***(M1)A1***

[4 marks]

b. area $= \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right|$ ***(M1)***

$= \frac{1}{2}\sqrt{4^2 + 2^2 + 2^2} = \frac{1}{2}\sqrt{24} \left(= \sqrt{6}\right)$ ***A1***

Note: Award ***M0A0*** for attempts that do not involve the answer to (a).

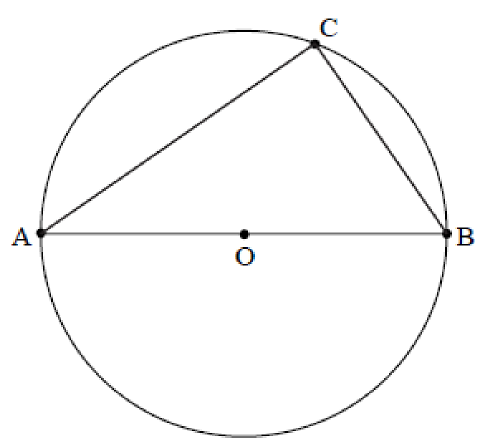
[2 marks]

Examiners report

a. Candidates showed a good understanding of the vector techniques required in this question.

b. Candidates showed a good understanding of the vector techniques required in this question.

In the diagram below, [AB] is a diameter of the circle with centre O. Point C is on the circumference of the circle. Let $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{c}$.



a. Find an expression for $\overrightarrow{CB}$ and for $\overrightarrow{AC}$ in terms of $\boldsymbol{b}$ and $\boldsymbol{c}$. [2]

b. Hence prove that $\mathrm{A\hat{C}B}$ is a right angle. [3]

Markscheme

a. $\overrightarrow{CB} = \boldsymbol{b} - \boldsymbol{c}$, $\overrightarrow{AC} = \boldsymbol{b} + \boldsymbol{c}$ ***A1A1***

Note: Condone absence of vector notation in (a).

[2 marks]

b. $\overrightarrow{AC} \cdot \overrightarrow{CB} = (\boldsymbol{b} + \boldsymbol{c}) \cdot (\boldsymbol{b} - \boldsymbol{c})$ ***M1***

$= |\boldsymbol{b}|^2 - |\boldsymbol{c}|^2$ ***A1***

$= 0$ since $|\boldsymbol{b}| = |\boldsymbol{c}|$ ***R1***

Note: Only award the ***A1*** and ***R1*** if working indicates that they understand that they are working with vectors.

so $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{CB}$ *i.e.* $A\hat{C}B$ is a right angle ***AG***

[3 marks]

Examiners report

a. Most candidates were able to find the expressions for the two vectors although a number were not able to do this. Most then tried to use Pythagoras' theorem and confused scalars and vectors. There were few correct responses to the second part. Candidates did not seem to be able to use the algebra of vectors comfortably.

b. Most candidates were able to find the expressions for the two vectors although a number were not able to do this. Most then tried to use Pythagoras' theorem and confused scalars and vectors. There were few correct responses to the second part. Candidates did not seem to be able to use the algebra of vectors comfortably.

Find the coordinates of the point of intersection of the planes defined by the equations $x + y + z = 3,\ x - y + z = 5$ and $x + y + 2z = 6$.

Markscheme

METHOD 1

for eliminating one variable from two equations ***(M1)***

eg, $\begin{cases} (x + y + z = 3) \\ 2x + 2z = 8 \\ 2x + 3z = 11 \end{cases}$ ***A1A1***

for finding correctly one coordinate

eg, $\begin{cases} (x + y + z = 3) \\ (2x + 2z = 8) \\ z = 3 \end{cases}$ ***A1***

for finding correctly the other two coordinates ***A1***

$\Rightarrow \begin{cases} x = 1 \\ y = -1 \\ z = 3 \end{cases}$

the intersection point has coordinates $(1,\ -1,\ 3)$

METHOD 2

for eliminating two variables from two equations or using row reduction ***(M1)***

eg, $\begin{cases} (x + y + z = 3) \\ -2 = 2 \\ z = 3 \end{cases}$ **or** $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ ***A1A1*** for finding correctly the other coordinates ***A1A1***

$$\Rightarrow \begin{cases} x = 1 \\ y = -1 \\ (z = 3) \end{cases} \text{ or } \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

the intersection point has coordinates $(1,\ -1,\ 3)$

METHOD 3

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -2$$ ***(A1)***

attempt to use Cramer's rule ***M1***

$$x = \frac{\begin{vmatrix} 3 & 1 & 1 \\ 5 & -1 & 1 \\ 6 & 1 & 2 \end{vmatrix}}{-2} = \frac{-2}{-2} = 1$$ ***A1***

$$y = \frac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 5 & 1 \\ 1 & 6 & 2 \end{vmatrix}}{-2} = \frac{2}{-2} = -1$$ ***A1***

$$z = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 1 & -1 & 5 \\ 1 & 1 & 6 \end{vmatrix}}{-2} = \frac{-6}{-2} = 3$$ ***A1***

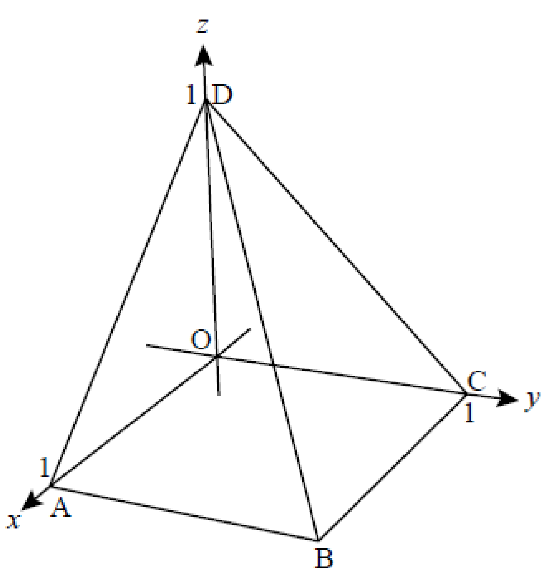
Note: Award ***M1*** only if candidate attempts to determine at least one of the variables using this method.

[5 marks]

Examiners report

[N/A]

The following figure shows a square based pyramid with vertices at O(0, 0, 0), A(1, 0, 0), B(1, 1, 0), C(0, 1, 0) and D(0, 0, 1).



The Cartesian equation of the plane Π_2, passing through the points B , C and D , is $y + z = 1$.

The plane Π_3 passes through O and is normal to the line BD.

Π_3 cuts AD and BD at the points P and Q respectively.

a. Find the Cartesian equation of the plane Π_1, passing through the points A , B and D. [3]

b. Find the angle between the faces ABD and BCD. [4]

c. Find the Cartesian equation of Π_3. [3]

d. Show that P is the midpoint of AD. [4]

e. Find the area of the triangle OPQ. [5]

Markscheme

a. recognising normal to plane or attempting to find cross product of two vectors lying in the plane ***(M1)***

for example, $\vec{\text{AB}} \times \vec{\text{AD}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ***(A1)***

$\Pi_1 : x + z = 1$ ***A1***

[3 marks]

b. **EITHER**

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 1 = \sqrt{2}\sqrt{2}\cos\theta$ ***M1A1***

OR

$\left| \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right| = \sqrt{3} = \sqrt{2}\sqrt{2}\sin\theta$ ***M1A1***

Note: ***M1*** is for an attempt to find the scalar or vector product of the two normal vectors.

$\Rightarrow \theta = 60^\circ \left(= \frac{\pi}{3}\right)$ ***A1***

angle between faces is $20^\circ \left(= \frac{2\pi}{3}\right)$ ***A1***

[4 marks]

c. $\vec{\text{DB}} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ or $\vec{\text{BD}} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ ***(A1)***

$\Pi_3 : x + y - z = k$ ***(M1)***

$\Pi_3 : x + y - z = 0$ ***A1***

[3 marks]

d. **METHOD 1**

line AD:(**r** =)$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ ***M1A1*** intersects Π_3 when $\lambda - (1 - \lambda) = 0$ ***M1***

so $\lambda = \frac{1}{2}$ ***A1***

hence P is the midpoint of AD ***AG***

METHOD 2

midpoint of AD is (0.5, 0, 0.5) ***(M1)A1***

substitute into $x + y - z = 0$ ***M1***

0.5 + 0.5 – 0.5 = 0 ***A1***

hence P is the midpoint of AD ***AG***

[4 marks]

e. **METHOD 1**

$\mathrm{OP} = \frac{1}{\sqrt{2}},\ \mathrm{O}\hat{\mathrm{P}}\mathrm{Q} = 90^\circ,\ \mathrm{O}\hat{\mathrm{Q}}\mathrm{P} = 60^\circ$ ***A1A1A1***

$\mathrm{PQ} = \frac{1}{\sqrt{6}}$ ***A1***

area $= \frac{1}{2\sqrt{12}} = \frac{1}{4\sqrt{3}} = \frac{\sqrt{3}}{12}$ ***A1***

METHOD 2

line BD:(**r** =) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

$\Rightarrow \lambda = \frac{2}{3}$ ***(A1)***

$\vec{\mathrm{OQ}} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$ ***A1***

area = $\frac{1}{2}\left|\vec{\mathrm{OP}} \times \vec{\mathrm{OQ}}\right|$ ***M1***

$\vec{\mathrm{OP}} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{pmatrix}$ ***A1***

Note: This ***A1*** is dependent on ***M1***.

area = $\frac{\sqrt{3}}{12}$ ***A1***

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

PQRS is a rhombus. Given that $\overrightarrow{PQ} = \boldsymbol{a}$ and $\overrightarrow{QR} = \boldsymbol{b}$,

(a) express the vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$;

(b) hence show that the diagonals in a rhombus intersect at right angles.

Markscheme

(a) $\overrightarrow{PR} = \mathbf{a} + \boldsymbol{b}$ ***A1***

$\overrightarrow{QS} = \boldsymbol{b} - \boldsymbol{a}$ ***A1***

[2 marks]

(b) $\overrightarrow{PR} \cdot \overrightarrow{QS} = (\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{b} - \boldsymbol{a})$ ***M1***

$= |\boldsymbol{b}|^2 - |\boldsymbol{a}|^2$ ***A1***

for a rhombus $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***R1***

hence $|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2 = 0$ ***A1***

Note: Do not award the final ***A1*** unless ***R1*** is awarded.

hence the diagonals intersect at right angles ***AG***

[4 marks]

Total [6 marks]

Examiners report

[N/A]

The vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ satisfy the equation $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \mathbf{0}$. Show that $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c} = \boldsymbol{c} \times \boldsymbol{a}$.

Markscheme

taking cross products with $\boldsymbol{a}$, ***M1***

$\boldsymbol{a} \times (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{0} = \boldsymbol{0}$ ***A1***

using the algebraic properties of vectors and the fact that $\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{0}$, ***M1***

$\boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c} = \boldsymbol{0}$ ***A1***

$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{c} \times \boldsymbol{a}$ ***AG***

taking cross products with $\boldsymbol{b}$, ***M1***

$\boldsymbol{b} \times (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{0}$

$\boldsymbol{b} \times \boldsymbol{a} + \boldsymbol{b} \times \boldsymbol{c} = \boldsymbol{0}$ ***A1***

$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c}$ ***AG*** this completes the proof

[6 marks]

Examiners report

[N/A]

---

a. Consider the vectors $\boldsymbol{a} = 6\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 4\boldsymbol{k}$. [11]

(i) Find the cosine of the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Find $\boldsymbol{a} \times \boldsymbol{b}$.

(iii) **Hence** find the Cartesian equation of the plane $\prod$ containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ and passing through the point $(1, 1, -1)$.

(iv) The plane $\prod$ intersects the x-y plane in the line l. Find the area of the finite triangular region enclosed by l, the x-axis and the y-axis.

b. Given two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, [8]

(i) show that $\boldsymbol{p} \cdot \boldsymbol{p} = |\boldsymbol{p}|^2$;

(ii) hence, or otherwise, show that $|\boldsymbol{p} + \boldsymbol{q}|^2 = |\boldsymbol{p}|^2 + 2\boldsymbol{p} \cdot \boldsymbol{q} + |\boldsymbol{q}|^2$;

(iii) deduce that $|\boldsymbol{p} + \boldsymbol{q}| \le |\boldsymbol{p}| + |\boldsymbol{q}|$.

Markscheme

a. (i) use of $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$ ***(M1)***

$\boldsymbol{a} \cdot \boldsymbol{b} = -1$ ***(A1)***

$|\boldsymbol{a}| = 7$, $|\boldsymbol{b}| = 5$ ***(A1)***

$\cos\theta = -\frac{1}{35}$ ***A1***

(ii) the required cross product is

$$\begin{vmatrix} i & j & k \\ 6 & 3 & 2 \\ 0 & -3 & 4 \end{vmatrix} = 18\boldsymbol{i} - 24\boldsymbol{j} - 18\boldsymbol{k}$$ ***M1A1***

(iii) using $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{p} \cdot \boldsymbol{n}$ the equation of the plane is ***(M1)***

$18x - 24y - 18z = 12 \quad (3x - 4y - 3z = 2)$ ***A1***

(iv) recognizing that $z = 0$ ***(M1)***

x-intercept $= \frac{2}{3}$, y-intercept $= -\frac{1}{2}$ ***(A1)***

area $= \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$ ***A1***

[11 marks]

b. (i) $\boldsymbol{p}\cdot\boldsymbol{p} = |\boldsymbol{p}||\boldsymbol{p}|\cos 0$ ***M1A1***

$= |\boldsymbol{p}|^2$ ***AG***

(ii) consider the LHS, and use of result from part (i)

$|\boldsymbol{p}+\boldsymbol{q}|^2 = (\boldsymbol{p}+\boldsymbol{q})\cdot(\boldsymbol{p}+\boldsymbol{q})$ ***M1***

$= \boldsymbol{p}\cdot\boldsymbol{p} + \boldsymbol{p}\cdot\boldsymbol{q} + \boldsymbol{q}\cdot\boldsymbol{p} + \boldsymbol{q}\cdot\boldsymbol{q}$ ***(A1)***

$= \boldsymbol{p}\cdot\boldsymbol{p} + 2\boldsymbol{p}\cdot\boldsymbol{q} + \boldsymbol{q}\cdot\boldsymbol{q}$ ***A1***

$= |\boldsymbol{p}|^2 + 2\boldsymbol{p}\cdot\boldsymbol{q} + |\boldsymbol{q}|^2$ ***AG***

(iii) **EITHER**

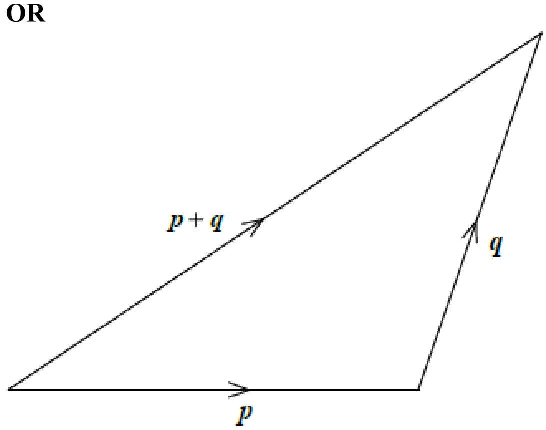
use of $\boldsymbol{p}\cdot\boldsymbol{q} \leqslant |\boldsymbol{p}||\boldsymbol{q}|$ ***M1***

so $0 \leqslant |\boldsymbol{p}+\boldsymbol{q}|^2 = |\boldsymbol{p}|^2 + 2\boldsymbol{p}\cdot\boldsymbol{q} + |\boldsymbol{q}|^2 \leqslant |\boldsymbol{p}|^2 + 2\,|\boldsymbol{p}||\boldsymbol{q}| + |\mathrm{q}|^2$ ***A1***

take square root (of these positive quantities) to establish ***A1***

$|\boldsymbol{p}+\boldsymbol{q}| \leqslant |\boldsymbol{p}| + |\boldsymbol{q}|$ ***AG***

OR



M1M1

Note: Award ***M1*** for correct diagram and ***M1*** for correct labelling of vectors including arrows.

since the sum of any two sides of a triangle is greater than the third side,

$|\boldsymbol{p}| + |\boldsymbol{q}| > |\boldsymbol{p}+\boldsymbol{q}|$ ***A1***

when $\boldsymbol{p}$ and $\boldsymbol{q}$ are collinear $|\boldsymbol{p}| + |\boldsymbol{q}| = |\boldsymbol{p}+\boldsymbol{q}|$

$\Rightarrow |\boldsymbol{p}+\boldsymbol{q}| \leqslant |\boldsymbol{p}| + |\boldsymbol{q}|$ ***AG***

[8 marks]

Examiners report

a. This was the most accessible question in section B for the candidates. The majority of candidates produced partially correct answers to part (a), with nearly all candidates being able to use the scalar and vector product. Candidates found part (iv) harder and often did not appreciate the significance of letting $z = 0$. Candidates clearly found part (b) harder and again this was a point where candidates lost time. Many candidates attempted this using components, which was fine in part (i), fine, but time consuming in part (ii), and extremely complicated in part (iii). A number of candidates lost marks because they were careless in showing their working in part (ii) which required them to “show that”.

b. This was the most accessible question in section B for the candidates. The majority of candidates produced partially correct answers to part (a), with nearly all candidates being able to use the scalar and vector product. Candidates found part (iv) harder and often did not appreciate the significance of letting $z = 0$. Candidates clearly found part (b) harder and again this was a point where candidates lost time. Many candidates attempted this using components, which was fine in part (i), fine, but time consuming in part (ii), and extremely complicated in part (iii). A number of candidates lost marks because they were careless in showing their working in part (ii) which required them to "show that".

(a) Show that a Cartesian equation of the line, l_1, containing points A$(1, -1, 2)$ and B$(3, 0, 3)$ has the form $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{1}$.

(b) An equation of a second line, l_2, has the form $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$. Show that the lines l_1 and l_2 intersect, and find the coordinates of their point of intersection.

(c) Given that direction vectors of l_1 and l_2 are $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ respectively, determine $\boldsymbol{d}_1 \times \boldsymbol{d}_2$.

(d) Show that a Cartesian equation of the plane, $\prod$, that contains l_1 and l_2 is $-x - y + 3z = 6$.

(e) Find a vector equation of the line l_3 which is perpendicular to the plane $\prod$ and passes through the point T(3, 1, −4).

(f) (i) Find the point of intersection of the line l_3 and the plane $\prod$.

(ii) Find the coordinates of T′, the reflection of the point T in the plane $\prod$.

(iii) Hence find the magnitude of the vector $\overrightarrow{\mathrm{TT'}}$.

Markscheme

(a) identifies a direction vector *e.g.* $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{\mathrm{BA}} = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}$ ***A1***

identifies the point (1, –1, 2) ***A1***

line l_1 : $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{1}$ ***AG***

[2 marks]

(b) $r = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $r = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$1 + 2\lambda = 1 + \mu, \ -1 + \lambda = 2 + 2\mu, \ 2 + \lambda = 3 + \mu$ ***(M1)***

equating two of the three equations gives $\lambda = -1$ and $\mu = -2$ ***A1A1***

check in the third equation

satisfies third equation therefore the lines intersect ***R1***

therefore coordinates of intersection are (–1, –2, 1) ***A1***

[5 marks]

(c) $\boldsymbol{d}_1 = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}, \boldsymbol{d}_2 = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ ***A1***

$\boldsymbol{d}_1 \times \boldsymbol{d}_2 = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ ***M1A1***

Note: Accept scalar multiples of above vectors.

[3 marks]

(d) equation of plane is $-x - y + 3z = k$ ***M1A1***

contains (1, 2, 3) (or $(-1,\ -2,\ 1)$ or $(1,\ -1,\ 2)$) $\therefore k = -1 - 2 + 3 \times 3 = 6$ ***A1***

$-x - y + 3z = 6$ ***AG***

[3 marks]

(e) direction vector of the perpendicular line is $\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$ ***(M1)***

$r = \begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} + m \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$ ***A1***

Note: Award ***A0*** if ***r*** omitted.

[2 marks]

(f) (i) find point where line meets plane

$-(3 - m) - (1 - m) + 3(-4 + 3m) = 6$ ***M1***

$m = 2$ ***A1***

point of intersection is (1, –1, 2) ***A1***

(ii) for $\mathrm{T}', m = 4$ ***(M1)***

so $\mathrm{T}' = (-1, -3, 8)$ ***A1***

(iii) $\overrightarrow{\mathrm{TT}'} = \sqrt{(3+1)^2 + (1+3)^2 + (-4-8)^2}$ ***(M1)***

$= \sqrt{176} \quad (= 4\sqrt{11})$ ***A1***

[7 marks]

Total [22 marks]

Examiners report

This question was done very well by many students. The common errors were using the same variable for line 2 and in stating the vectors in b) were not parallel and therefore the lines did intersect. Many students did not check the solution in order to establish this.

When required to give the equation of the line in e) many did not state it as an equation, let alone a vector equation.

The difference between position vectors and coordinates was not clear on many papers.

In f) many used inefficient techniques that were time consuming to find the point of reflection.

The acute angle between the vectors 3***i*** – 4***j*** – 5***k*** and 5***i*** – 4***j*** + 3***k*** is denoted by *θ*.

Find cos *θ*.

Markscheme

$\cos\theta = \frac{(3i-4j-5k)\bullet(5i-4j+3k)}{|3i-4j-5k||5i-4j+3k|}$ ***(M1)***

$= \frac{16}{\sqrt{50}\sqrt{50}}$ ***A1A1***

Note: ***A1*** for correct numerator and ***A1*** for correct denominator.

$= \frac{8}{25}\left(= \frac{16}{50} = 0.32\right)$ ***A1***

[4 marks]

Examiners report

[N/A]

Let α be the angle between the unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, where $0 \leqslant \alpha \leqslant \pi$.

(a) Express $|\boldsymbol{a} - \boldsymbol{b}|$ and $|\boldsymbol{a} + \boldsymbol{b}|$ in terms of α.

(b) Hence determine the value of $\cos\alpha$ for which $|\boldsymbol{a} + \boldsymbol{b}| = 3\,|\boldsymbol{a} - \boldsymbol{b}|$.

Markscheme

METHOD 1

(a) $|\boldsymbol{a} - \boldsymbol{b}| = \sqrt{|a|^2 + |b|^2 - 2\,|a|\,|b|\cos\alpha}$ ***M1***

$= \sqrt{2 - 2\cos\alpha}$ ***A1***

$|\boldsymbol{a} + \boldsymbol{b}| = \sqrt{|a|^2 + |b|^2 - 2\,|a|\,|b|\cos(\pi - \alpha)}$

$= \sqrt{2 + 2\cos\alpha}$ ***A1***

Note: Accept the use of a, b for $|\boldsymbol{a}|$, $|\boldsymbol{b}|$.

(b) $= \sqrt{2 + 2\cos\alpha} = 3\sqrt{2 - 2\cos\alpha}$ ***M1***

$\cos\alpha = \frac{4}{5}$ ***A1***

METHOD 2

(a) $|\boldsymbol{a} - \boldsymbol{b}| = 2\sin\frac{\alpha}{2}$ ***M1A1***

$|\boldsymbol{a} + \boldsymbol{b}| = 2\sin\left(\frac{\pi}{2} - \frac{\alpha}{2}\right) = 2\cos\frac{\alpha}{2}$ ***A1***

Note: Accept the use of a, b for $|\boldsymbol{a}|$, $|\boldsymbol{b}|$.

(b) $2\cos\frac{\alpha}{2} = 6\sin\frac{\alpha}{2}$

$\tan\frac{\alpha}{2} = \frac{1}{3} \Rightarrow \cos^2\frac{\alpha}{2} = \frac{9}{10}$ ***M1***

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 = \frac{4}{5}$ ***A1***

[5 marks]

Examiners report

To solve this problem, candidates had to know either that $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}+\boldsymbol{b}) = |\boldsymbol{a}+\boldsymbol{b}|^2$ or that the diagonals of a parallelogram whose sides are $\boldsymbol{a}$ and $\boldsymbol{b}$ represent the vectors $\boldsymbol{a}+\boldsymbol{b}$ and $\boldsymbol{a}-\boldsymbol{b}$. It was clear from the scripts that many candidates were unaware of either result and were therefore unable to make any progress in this question.

A triangle has vertices A(1, –1, 1), B(1, 1, 0) and C(–1, 1, –1) .

Show that the area of the triangle is $\sqrt{6}$.

Markscheme

METHOD 1

for finding two of the following three vectors (or their negatives)

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}, \overrightarrow{\mathrm{BC}} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$ ***(A1)(A1)***

and calculating

EITHER

$\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}} = \begin{vmatrix} i & j & k \\ 0 & 2 & -1 \\ -2 & 2 & -2 \end{vmatrix} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$ ***M1A1***

area $\Delta\mathrm{ABC} = \frac{1}{2}\left|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}\right|$ ***M1***

OR

$\overrightarrow{\mathrm{BA}} \times \overrightarrow{\mathrm{BC}} = \begin{vmatrix} i & j & k \\ 0 & -2 & 1 \\ -2 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ ***M1A1***

area $\Delta\mathrm{ABC} = \frac{1}{2}\left|\overrightarrow{\mathrm{BA}} \times \overrightarrow{\mathrm{BC}}\right|$ ***M1***

OR

$\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}} = \begin{vmatrix} i & j & k \\ 2 & -2 & 2 \\ 2 & 0 & 1 \end{vmatrix} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$ ***M1A1***

area $\Delta\mathrm{ABC} = \frac{1}{2}\left|\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}\right|$ ***M1***

THEN

area $\Delta\mathrm{ABC} = \frac{\sqrt{24}}{2}$ ***A1***

$= \sqrt{6}$ ***AG*** ***N0***

METHOD 2

for finding two of the following three vectors (or their negatives)

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}, \overrightarrow{\mathrm{BC}} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$ ***(A1)(A1)***

EITHER

$\cos A = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|}$ ***M1***

$= \frac{6}{\sqrt{5}\sqrt{12}} = \frac{6}{\sqrt{60}} \left(\text{or } \frac{3}{\sqrt{15}}\right)$

$\sin A = \sqrt{\frac{2}{5}}$ ***A1***

area $\Delta\text{ABC} = \frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin A$ ***M1***

$= \frac{1}{2}\sqrt{5}\sqrt{12}\sqrt{\frac{2}{5}}$

$= \frac{1}{2}\sqrt{24}$ ***A1***

$= \sqrt{6}$ ***AG*** ***N0***

OR

$\cos B = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|}$ ***M1***

$= -\frac{1}{\sqrt{5}\sqrt{5}} = -\frac{1}{5}$

$\sin B = \sqrt{\frac{24}{25}} \left(\text{or } \frac{\sqrt{24}}{5}\right)$ ***A1***

area $\Delta\text{ABC} = \frac{1}{2}\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\sin B$ ***M1***

$= \frac{1}{2}\sqrt{5}\sqrt{5}\sqrt{\frac{24}{25}}$

$= \frac{1}{2}\sqrt{24}$ ***A1***

$= \sqrt{6}$ ***AG*** ***N0***

OR

$\cos C = \frac{\overrightarrow{CA} \cdot \overrightarrow{CB}}{\left|\overrightarrow{CA}\right|\left|\overrightarrow{CB}\right|}$ ***M1***

$= \frac{6}{\sqrt{12}\sqrt{5}} = \frac{6}{\sqrt{60}} \left(\text{or } \frac{3}{\sqrt{15}}\right)$

$\sin C = \sqrt{\frac{2}{5}}$ ***A1***

area $\Delta\text{ABC} = \frac{1}{2}\left|\overrightarrow{CA}\right|\left|\overrightarrow{CB}\right|\sin C$ ***M1***

$= \frac{1}{2}\sqrt{12}\sqrt{5}\sqrt{\frac{2}{5}}$

$= \frac{1}{2}\sqrt{24}$ ***A1***

$= \sqrt{6}$ ***AG*** ***N0***

METHOD 3

for finding two of the following three vectors (or their negatives)

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$ ***(A1)(A1)***

$\text{AB} = \sqrt{5} = c\,,\ \text{AC} = \sqrt{12} = 2\sqrt{3} = b\,,\ \text{BC} = \sqrt{5} = a$ ***M1A1***

$s = \frac{\sqrt{5}+2\sqrt{3}+\sqrt{5}}{2} = \sqrt{3} + \sqrt{5}$ ***M1***

area $\Delta\text{ABC} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(\sqrt{3}+\sqrt{5})(\sqrt{3})(\sqrt{5}-\sqrt{3})(\sqrt{3})}$

$= \sqrt{3(5-3)}$ ***A1***

$= \sqrt{6}$ ***AG*** ***N0***

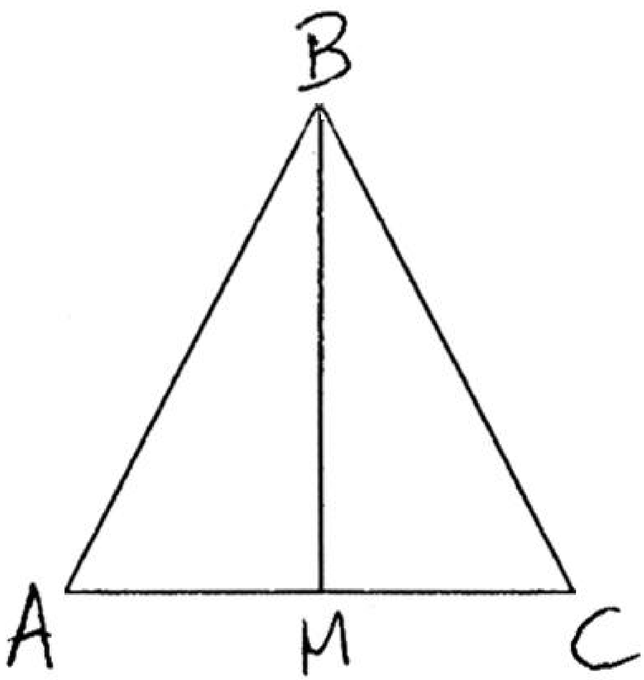
METHOD 4

for finding two of the following three vectors (or their negatives)

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$ ***(A1)(A1)***

$AB = BC = \sqrt{5}$ and $AC = \sqrt{12} = 2\sqrt{3}$ ***M1A1***

ΔABC is isosceles



let *M* be the midpoint of [AC] , the height $BM = \sqrt{5-3} = \sqrt{2}$ ***M1***

area $\Delta ABC = \frac{2\sqrt{3}\times\sqrt{2}}{2}$ ***A1***

$= \sqrt{6}$ ***AG*** ***N0***

[6 marks]

Examiners report

This question was answered fairly well by most candidates using a diversity of approaches.